PACIFIC JOURNAL OF MATHEMATICS
Vol. 27, No. 2, 1968

UNCOUNTABLY MANY ALMOST POLYHEDRAL WILD $(k-2)$-CELLS IN E^k FOR $k \geqq 4$

LESLIE C. GLASER

In [1] infinitely many almost polyhedral wild arcs were constructed in E^3 so as to have an end point as the "bad' point. In [5] uncountably many almost polyhedral wild arcs were constructed in E^3 with an interior point as the "bad" point. In [4] Doyle and Hocking constructed an almost polyhedral wild disk in E^4 with the property that the proof of the nontameness is perhaps the most elementary possible. They state that essentially the same construction yields a wild $(n-2)$-disk in E^n for $n \geqq 4$. Here, making use of the construction given in [4], we prove that for each $k \geqq 4$, there exist uncountably many almost polyhedral wild $(k-2)$-cells in E^k. To obtain the above result we also prove that for each $k \geqq 3$, there exist countably many polyhedral locally flat $(k-2)$-spheres in E^k so that the fundamental groups of the complements of these spheres are all distinct and given any two of these groups, one is not the surjective image of the other.

A set S in E^k is polyhedral if it can be covered by a finite rectilinear subcomplex of E^k. A $(k-2)$-cell D in E^k is almost polyhedral if for some point $q \in D$, $D - \{q\}$ can be covered by an infinite locally finite rectilinear subcomplex of $E^k - \{q\}$. The $(k-2)$-cells constructed here all have $q \in \mathrm{Bd}\, D$. D is wild if there does not exist a homeomorphism h of E^k onto itself such that $h(D)$ is a finite rectilinear subcomplex of E^k. An n-manifold $M^n \subset E^k$ is locally flat if each $p \in \mathrm{int}\, M (p \in \mathrm{Bd}\, M)$ has a neighborhood U in E^k such that the pair $(U, U \cap M)$ is homeomorphic as pairs to (E^k, E^n) (to (E^k, E^n_+)).

THEOREM 1. *There exist countably many polyhedral simple closed curves $\{J_n\}$ $(n = 1, 2, 3, \cdots)$ in E^3 so that if $G_n \cong \pi_1(E^3 - J_n)$, then for all positive integers n and m $(n \neq m)$, $G_n \not\cong Z$ and $G_n \not\cong G_m$. Furthermore, if $m > n$, then there is no surjection of G_m onto G_n.*

Proof. Expressing points of E^3 in terms of cylindrical coordinates (θ, r, z), let T be the "unknotted" torus $(r-2)^2 + z^2 = 1$. Let $K_{p,q}$ denote the torus knot of type p, q, where p and q are relatively prime nonnegative integers and $K_{p,q}$ is a curve on the surface T that cuts a merdian in p points and a longitude in q points. More precisely, $K_{p,q}$ is defined by the equations $r = 2 + \cos(q\theta/p)$ and $z = \sin(q\theta/p)$.