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CRITICAL POINTS ON RIM-COMPACT SPACES

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In this note we prove that all the points of a rim-compact space X at which X is not locally compact are critical points for any local dynamical system defined on X. When a local system is global this result is obtained by extending the global system π on X to a global system ρ on the Freudenthal compactification Y of X, then showing that Y-X is a critical set for ρ, and, finally, observing that $\overline{Y\text{-}X}$ contains all the points of X at which X is not locally compact. This weaker result will appear in the author's doctoral dissertation and requires the use of general extension theorems proven there. For this paper, we isolate those parts of the thesis which are pertinent to our theorem.

DEFINITION 1. [1]. A (continuous) local dynamical system on a topological space Z is an object π satisfying the following conditions: (R is the set of real numbers with the usual topology)

(1) π is a continuous partial map from $Z \times R$ into Z.

(2) For every z in Z there are (bounds) α_z and ω_z such that $-\infty \leqq \alpha_z < 0$, $0 < \omega_z \leqq +\infty$ and $\pi(z, t)$ is defined if and only if t is in (α_z, ω_z).

(3) The domain of π is open in $Z \times R$.

(4) For each z in Z, $\pi(z, 0) = z$.

(5) When $\pi(z, t)$ is defined, $\pi(\pi(z, t), t') = \pi(z, t+t')$ whenever either side is defined.

A local dynamical system π on Z is global if $\alpha_z = -\infty$ and $\omega_z = +\infty$ for all $z \in Z$. Condition (4) is called the initial value condition, and condition (5) the additive condition. Because of the additive condition, it has become conventional to write $\pi(z, t)$ as $z\pi t$. Thus the equality in condition (5) is written $(z\pi t)\pi t' = z\pi(t + t')$. To prove that a point z is a critical point of π, it suffices to show there is an $\varepsilon > 0$ such that $z = z\pi t$ for all $t \in [0, \varepsilon)$.

DEFINITION 2. A topological space X is said to be rim-compact if and only if it is T_2 and each point of X has a fundamental system of neighborhoods with compact boundaries.

In this paper X denotes a rim-compact space and Y denotes the Freudenthal compactification of X, ([2], p. 111). Y is T_2 and every point in Y has a fundamental system of neighborhoods with compact boundaries entirely in X. These and compactness are the only properties of Y that will be used. Convergence of a net $\{y_i\}$ indexed