

ARTINIAN, ALMOST ABELIAN GROUPS AND THEIR GROUPS OF AUTOMORPHISMS

ANNEMARIE SCHLETTE

The principal results of this paper are several characterizations of the class of artinian almost abelian groups. Several corollaries of the main characterization are given, one of which states that a torsion group G which has a finite group of automorphisms is finite, and an abelian torsion group which has a countable automorphism group is finite. In a concluding section, the class of artinian almost abelian groups and the class of finite groups are characterized by formal group-theoretical properties.

A group is called artinian, if the minimum condition is satisfied by its subgroups. The structure of an abelian, artinian group is completely determined. Every artinian group that is known so far is an extension of an abelian, artinian group by a finite group. An extension of an abelian group by a finite group is called almost abelian. It is not known whether there exist further artinian groups. For this reason it seems to be worth while to consider the structure of artinian and almost abelian groups and we will give some characterizations of them. We will prove the following

MAIN THEOREM. *The following properties of the group G are equivalent:*

- (1) G is artinian and almost abelian.
- (2) (a) Every abelian subnormal subgroup of G is artinian;
(b) Every infinite epimorphic image of G possesses an abelian subnormal subgroup, not 1;
(c) G is a torsion group.
- (3) (a) Every abelian subgroup of G is artinian;
(b) Every epimorphic image, not 1, of G possesses an almost abelian normal subgroup, not 1.
- (4) (a) G is a torsion group;
(b) Every torsion group of automorphisms of G is artinian and almost abelian.
- (5) (a) G is a torsion group;
(b) The central quotient group of G is artinian and almost abelian;
(c) Primary elementary abelian groups of automorphisms of G are countable.

Sections 1 to 4 deal with preliminary lemmas, some of which may