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A REPRESENTATION THEOREM FOR MEASURES ON INFINITE DIMENSIONAL SPACES

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If X is a locally compact, regular topological space, then the well known Riesz representation theorem sets up an isomorphism between the family of all bounded Radón outer measures on X and the set of continuous positive linear functionals on the family of continuous functions with compact support in X. In this isomorphism corresponding elements, l a linear functional and μ a measure, satisfy the relationship $l(f) = \int f d\mu$ for all continuous functions f with compact support in X.

Since an infinite product of locally compact, regular spaces is in general no longer locally compact with respect to the product topology, the Riesz representation theorem fails to hold for such spaces. In this paper, an analogue of the Riesz representation theorem is obtained for this case.

The main idea is to replace the various families mentioned above by the following:

(i) A family $\mathscr{C}$ of cylinders whose elements act like compact sets for a "pseudo-topology" $\mathscr{G}$, where $\mathscr{G}$ is closed under finite intersections and countable unions and is a subset of the product topology.

(ii) A family M of bounded outer measures, related to $\mathscr{C}$ and $\mathscr{G}$ in much the same way as bounded Radón outer measures are related to compact and open sets.

(iii) A family F of functions depending only on a finite number of coordinates, with respect to which they are continuous and have compact support.

(iv) A family L of positive linear functionals on the linear span of F.

Under the added hypothesis of σ-compactness of the coordinate spaces, we show that L and M are isomorphic in such a way that corresponding elements, l in L and μ in M, satisfy the relationship $l(f) = \int f d\mu$ for all f in F.

Moreover we show that the elements of M can be viewed as the projective limit measures of projective systems of bounded regular Borel measures.

From the integrability of the members of F, it follows that all bounded Borel functions which depend only on a finite number of coordinates are also integrable. Thus the simple functions used by Šilov [7] and the tame functions used by Segal [6] and Gross [2] in the development of an