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ON CONTINUOUS MAPPINGS OF METACOMPACT ČECH COMPLETE SPACES

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Under what may be thought of as a guise of a description of pathology are indicated here certain ways in which Čech completeness, Arhangel'skiĭ's p-space concept, and metacompactness enlarge on the respective concepts of metric absolute G_δ's, metrizability, and paracompactness. This is done through examination of certain aspects of the theory of multivalued mappings. It is taken as a point of orientation that the topic of Tychonoff locally bicompact spaces has a substantial mathematical interest. It is assumed obvious that such spaces are locally paracompact p-spaces. An underlying point of view is that the class of regular locally paracompact p-spaces extends along natural lines the class of regular locally metrizable spaces.

Let us observe these theorems: (1) A Hausdorff space is paracompact if and only if it is fully normal [13]. (2) A space is metacompact if and only if for every collection G of open sets covering it there exists a collection H of open sets covering it such that if P is a point, the collection of all members of H containing P refines a finite subcollection K of G [22]. (3) A T_1 space is metrizable if and only if it is fully normal and has a base of countable order (cf. definitions below) [3]. (4) A T_1 space has a uniform base (cf. definitions below) if and only if it is metacompact and has a base of countable order [27]. (5) Metacompactness is invariant under the action on a topological space of a closed continuous mapping [23]. We may then see that whether or not a metacompact T_1 topological space S has a perfect mapping onto a space with a uniform base depends only on whether S has a perfect mapping onto a space having a base of countable order. Similarly, since full normalcy of a topological space is also an invariant under the action of a closed continuous mapping [10], whether a fully normal T_1 space S has a perfect mapping onto a metrizable space depends only on whether S has a perfect mapping onto a space having a base of countable order. These reductions achieve heightened interest in view of the invariance of the base of countable order property under the actions of peripherally bicompact closed continuous mappings on T_1 spaces [21], the intimacy of its relation to the topic of interior mappings [16, 19], and certain work of Frolík and Arhangel'skiĭ which will now be described.

Frolík showed that a paracompact Hausdorff space is Čech complete (cf. definition below) if and only if it has a perfect mapping