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MARTINGALES OF VECTOR VALUED SET FUNCTIONS

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This paper is concerned with the norm convergence of Banach space valued martingales in Orlicz spaces whose underlying measure is (possibly) only finitely additive. Because of the possible incompleteness of these Orlicz spaces of measurable point functions, this subject will be treated in the setting of Orlicz spaces of set functions V^{Φ} rather than the corresponding spaces L^{Φ} of measurable point functions. First, a conditional expectation P_B, operating on finitely additive set functions, is introduced and related to the usual conditional expectation E^B operating on L^1 by the equality

$$(*) \qquad P_B(F)(E) = \int_E E^B(f)d\mu \qquad E \in \Sigma$$

where (Ω, Σ, μ) is a measure space, B is a sub σ-field of Σ and $F(E) = \int_E fd\mu$ for $E \in \Sigma$.

Then, with the use of P_B martingales of set functions are defined and their convergence in appropriate V^{Φ} spaces is investigated. In addition, in the countably additive case, the results obtained for martingales of set functions are related to martingales of measurable point functions and extensions of certain results of Scalora, Chatterji, and Helms are obtained.

The study of finitely additive set functions appears to have begun during the close of the last century with such notions as Jordan Content. Through the first half of this century, with the introduction of the Lebesgue theory, most effort was concentrated on countably additive set functions. Recently, however, certain work, such as representations of linear functionals of the space of bounded functions has demanded the employment of finitely additive set functions. More important is the fact that finitely additive set functions provide considerable flexibility in applications and are sometimes no more untractable than their countably additive counterparts.

In their new approach to probability theory, Dubins and Savage [6] have noted that countable additivity is sometimes unnecessarily restrictive and have dropped it. In the study of the classical function spaces L^p, Bochner [1] and Leader [12] find it "natural to consider" the L^p spaces of finitely additive set functions. More recently in [16, 17] Bochner and Leader's groundwork was placed in the Orlicz space setting. In various ways, each of these papers present the argument