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SEMI-GROUPS OF SCALAR TYPE OPERATORS IN BANACH SPACES

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This paper deals with the spectral representation theorems of semi-groups of scalar type operators in Banach spaces. These results generalize the corresponding ones on semi-groups of hermitian, normal and unitary operators in Hilbert spaces. In the beginning sections we study some interesting properties of a $W^*(||\cdot||)$-algebra-which generalizes the notion of an abelian von Neumann algebra to Banach spaces-and unbounded spectral operators arising out of $E(\cdot)$-unbounded measurable functions where $E(\cdot)$ is a resolution of the identity. These results are applied later to prove the spectral representation theorems on semi-groups of scalar type operators. The last theorem of this paper gives an extension of Stone's theorem on strongly continuous one parameter group of unitary operators to arbitrary Banach spaces.

This paper mainly deals with the spectral representation theorems of semi-groups of scalar type operators in Banach spaces, generalizing those of semi-groups of hermitian, normal and unitary operators in Hilbert spaces. Since all the classical proofs of these theorems vitally depend on the inner-product structure of the Hilbert space they cannot be adapted to Banach spaces. However, Phillips has obtained in [15] these spectral representation theorems on Hilbert spaces by making use of the theory of abelian W^* (von Neumann) algebras. Here we adapt his method of proof by suitably generalizing the notion of an abelian W^* algebra to Banach spaces.

In [3] Bade has developed the theory of operator algebras W on Banach spaces, which are generated in the weak operator topology by a σ-complete Boolean algebra of projections. Such an algebra W has its maximal ideal space extremally disconnected, just as in the case of an abelian W^* algebra. However, W is not a B^*-algebra. To this end we exploit the work on hermitian operators in Banach spaces by Berkson [4, 5, 6], Lumer [12, 13] and Vidav [18] and we define an algebra called a $W^*(||\cdot||)$-algebra in § 2 of this paper, which is a B^*-algebra generated in the weak operator topology by a σ-complete Boolean algebra of projections. The involution * of this algebra is also strongly continuous (Theorem 1 of § 2) as its counterpart in the abelian W^*algebra. Thus $W^*(||\cdot||)$-algebras have all the essential properties of abelian W^* algebras, though the double commutant theorem fails for such algebras. (See Dieudonné [7]).

Further in § 2 we introduce an ordering relation among hermitian