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HOMOMORPHISMS OF ANNIHILATOR BANACH ALGEBRAS, II

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Let A be a semi-simple annihilator Banach algebra, and let ν be a homomorphism of A into a Banach algebra. In this paper it is shown that there exists a constant K and dense two-sided ideals containing the socle, I_L and I_R, such that $\|\nu(xy)\| \leqq K\|x\| \cdot \|y\|$ whenever $x \in I_L$ or $y \in I_R$. If A has a bounded left or right approximate identity, then ν is continuous on the socle. Thus if $A = L_1(G)$, where G is a compact topological group, then any homomorphism of A into a Banach algebra is continuous on the trigonometric polynomials.

In [1] we considered the problem of deducing continuity properties of a homomorphism ν from a semi-simple annihilator Banach algebra A into an arbitrary Banach algebra. The main theorem there (Theorem 5.1) had a conclusion more restrictive than the one stated above and required the additional hypothesis that $I \oplus \Re(I) = A$, for all closed two-sided ideals I, where $\Re(I) = \{x \mid Ix = (0)\}$. The main theorem of this paper applies when $A = L_p(G)$, $1 \leqq p < \infty$ or $C(G)$, where G is a compact topological group and multiplication is convolution, and when A is topologically-simple, whereas the earlier theorem did not.

Any terms not defined in this paper are those of Rickart's book [10]. For facts about annihilator algebras, the reader is referred to [4] or [10].

Given the left-right symmetry in the definition of annihilator algebras, it follows that, given any theorem about left (right) ideals, the corresponding theorem for right (left) ideals also holds. Specifically, this is the case for the theorems in [4, § 4] and [1, § 4]. We will make tacit use of this fact throughout this paper.

2. Structural lemmas. In this section several lemmas are established which will be used later in proving the main result. Throughout this section, we assume that A is a semi-simple annihilator Banach algebra.

LEMMA 2.1. *If $\{x_1, \cdots, x_n\}$ is contained in the socle of A, then there exist idempotents e and f such that $x_i \in eAf$, $1 \leqq i \leqq n$.*

Proof. By [1, Corollary 4.9], for each i there exist idempotents e_i and f_i such that $x_i \in e_iA \cap Af_i \subset e_iAf_i$. By [1, Th. 4.8], there