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DISTRIBUTION OF ZEROS OF SOLUTIONS OF A FOURTH ORDER DIFFERENTIAL EQUATION

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The primary concern of this paper is to study the distribution of zeros of solutions, that have at least four zeros, two or more of which are distinct zeros, of the canonical fourth order equation

$$(E_4) \qquad L_4[y] = (r_3 L_3[y])' + q_3 r_2 L_2[y] + q_4 y = 0\,,$$

where $r_i(x) > 0$, $r_i(x), q_j(x) \in C[a, \infty)$, $i = 1, 2, 3$, $j = 1, 2, 3, 4$, **which was introduced by Barrett.**

The canonical second order equation

$$(E_2) \qquad L_2[y] = (r_1 y')' + q_1 y = 0\,,$$

where $r_1(x) > 0$, $r_1(x), q_1(x) \in C\,[a, \infty)$, has been studied extensively. The canonical third order linear differential equation

$$(E_3) \qquad L_3[y] = (r_2 L_2[y])' + q_2(r_1 y') = 0\,,$$

where $r_i(x) > 0$, $r_i(x), q_i(x) \in C\,[a, \infty)$, $i = 1, 2$, which was introduced by Barrett, is a generalization of the second order equation (E_2).

Dolan studied the distribution of zeros of extremal solutions of (E_3) for the first conjugate point $\eta_1(t)$. In paragraph 2 the same study is made for the equation (E_4) and many of Dolan's ideas and techniques are used there. The results in paragraph 2 substantially complete the investigation begun in a paper by Aliev. Aliev defined and investigated the numbers $r_{ijk}(t)$ and $r_{1111}(t)$, which are extensions of the two-point nonoscillation numbers $r_{ij}(t)$ of Azbelev and Caljuk. Several of his results were reported in sources, which did not include the proofs, and these proofs were unavailable to the author, e.g.,

$$r_{31}(t) \geqq r_{211}(t) \geqq \min\,[r_{22}(t), r_{31}(t)]\,.$$

Aliev also proved that

$$r_{1111}(t) = \min\,[r_{121}(t), r_{112}(t)]\,,$$

and purported to prove that $r_{1111}(t) = \min\,[r_{211}(t), r_{112}(t)]$ but his proof is incorrect and this remains an open question. In paragraph 3 these results of Aliev are reproved and a much more complete picture is presented in the ordering of the numbers $\eta_1(t)$, $r_{ij}(t)$, $r_{ijk}(t)$, and $r_{1111}(t)$. The main results of this paper appear in paragraph 3.