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A CLASSIFICATION OF CENTERS

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The purpose of this paper is to classify centers according to isomorphisms. We define three types of isomorphism, and for two of these types give necessary and sufficient conditions for two centers to be isomorphic. We also give necessary and sufficient conditions for the third type of isomorphism to be equivalent to one of the other two.

These isomorphisms are discussed in a more general situation by Taro Ura [7]. This paper was motivated by discussions with Taro Ura and Otomar Hájek.

In our investigation we construct a section which generates a neighborhood of the center by using a theorem from the theory of fibre bundles. This section may be constructed directly, using the existence of a transversal through each noncritical point of the dynamical system. Much insight, which is otherwise lost, into the structure of a center is obtained from the fibre bundle approach.

The concept of a transversal is essential in our investigation. The basic material on transversal theory in planar dynamical systems is found in [3].

Throughout this paper R^+, R^1, and R^2 will denote the nonnegative reals, the reals, and the plane respectively.

Let (X, π) be a dynamical system on X, i.e., X is a topological space and π is a mapping of $X \times R^1$ onto X satisfying the following axioms (where $x\pi t = \pi(x, t)$ for $(x, t) \in X \times R^1$):

(1) Identity Axiom: $x\pi 0 = x$ for $x \in X$

(2) Homomorphism Axiom: $(x\pi t)\pi s = x\pi(t + s)$ for $x \in X$ and $t, s \in R^1$

(3) Continuity Axiom: π is continuous on $X \times R^1$.

Then for $x \in X$, $x\pi R^1$ is called the trajectory through x and is denoted by $C(x)$. If $C(x) = \{x\}$, x is called a critical point. If there exists $t \in R^1$, $t \neq 0$, such that $x\pi t = x$, $C(x)$ is called periodic. If $C(x)$ is periodic and x is not a critical point, $C(x)$, is called a cycle.

1. Definition and properties of a center.

In the following (R^2, π) will denote a dynamical system on R^2 and P the set of noncritical periodic points of (R^2, π). Let $T: P \to R^1$ be the mapping which associates with each point $x \in P$ its fundamental period $T(x)$. For the proof of the following result see [3, VII, 4.15].

PROPOSITION 1.1. *T is a continuous mapping of P into R^1.*