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DETERMINATION OF HYPERBOLICITY BY PARTIAL PROLONGATIONS

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In [3] it was shown that non-hyperbolic systems of partial differential equations may sometimes be altered by partial prolongations so they become hyperbolic. This paper solves two problems concerning this process for normal systems with two independent variables. First, if hyperbolicity is obtainable, it can be obtained after a bounded number of steps, the bound depending only on the algebraic structure of the given system and easily calculated. Second, an explicit procedure is described whereby any system which is absolutely equivalent to a hyperbolic system can be changed into a hyperbolic system. In addition much of the underlying algebraic structure of such systems and their partial prolongations is analyzed.

These problems are local in nature, so we generally use local coordinate expressions. Readers familiar with fibre bundle and jet terminology will easily see how to express many concepts in the language of modern differential geometry. *All functions are assumed infinitely differentiable.*

1. Systems. Let $\sum$ denote a system of quasi-linear partial differential equations in dependent variables $z^1, \cdots, z^m$ and two independent variables x^1, x^2 which can be solved for the partials $\partial z^\lambda/\partial x^2 = \partial_2 z^\lambda$:

$$\Sigma\begin{cases}\partial_2 z^\lambda = A^\lambda_\mu \partial_1 z^\mu + B^\lambda, \lambda = 1, \cdots, m\ ,\\ \Theta^\alpha = C^\alpha_\lambda \partial_1 z^\lambda + D^\alpha = 0, \alpha = 1, \cdots, \alpha_1\ ,\\ f^\beta(x^i, z^\lambda) = 0, \beta = 1, \cdots, \beta_1\ .\end{cases}$$

Here we use the summation convention, $\partial_1 z^\lambda = \partial z^\lambda/\partial z^1$, and $A^\lambda_\mu, B^\lambda, C^\alpha_\lambda, D^\alpha, f^\beta$ are (infinitely differentiable) functions of $x^1, x^2, z^1, \cdots, z^m$, on some open set U in R^{m+2}.

The following notation will be useful. If $F = F(x^1, x^2, z^1, \cdots, z^m, \partial_1 z^1, \cdots, \partial_1 z^m)$ is a function on an open set V of R^{2m+2}, then for $(x^1, x^2, z^1, \cdots, z^m)$ in U, we define

$$\begin{aligned}\partial_1^* F &= F_{x^1} + F_{z^\lambda}\partial_1 z^\lambda + F_{\partial_1 z^\lambda}\partial_{11} z^\lambda\ ,\\ \partial_2^* F &= F_{x^2} + F_{z^\lambda}[A^\lambda_\mu \partial_1 z^\mu + B^\lambda]\\ &\quad + F_{\partial_1 z^\lambda}\partial_1^*[A^\lambda_\mu \partial_1 z^\mu + B^\lambda]\ .\end{aligned}$$

These are functions of $x^1, x^2, z^1, \cdots, z^m, \partial_1 z^1, \cdots, \partial_1 z^m, \partial_{11} z^1, \cdots, \partial_{11} z^m$ where $\partial_{11} z^\lambda = \partial^2 z^\lambda/\partial x^1 \partial x^1$.