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CONJUGATE SURFACES FOR MULTIPLE INTEGRAL PROBLEMS IN THE CALCULUS OF VARIATIONS

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The Jacobi equation of the second variation for a multiple integral problem in the calculus of variations is a linear second order elliptic type partial differential equation provided certain hypotheses hold in the multiple integral problem. By means of the theory of quadratic forms in Hilbert space already present in the literature pertinent properties of solutions of such partial differential equations can be established. Here the pertinent property discussed is the vanishing of a solution on the boundary of a region, i.e. the existence of a conjugate surface of the differential equation. After developing the notion of focal point and stating the index theorems of the associated quadratic form, the existence of one parameter families of conjugate surfaces is shown, and illustrations of the theory are given.

1. Introduction. Fundamental theorems for quadratic forms in Hilbert space which are pertinent to problems in the calculus of variations are established in [7], [9] by Hestenes. Included in [7] is a theory of indices for an important class of quadratic forms arising in variational theory, and a general theory of focal points applicable to simple or multiple integral problems. Illustrations of the applications of focal point theory to one independent variable variational problems are given, as well as to boundary value problems for ordinary differential equations. The theory is, however, also applicable to multiple integral problems, and to boundary value problems for elliptic partial differential equations (indeed for integro-differential equations), and the author had this in mind in the formulation of the theory. In [9] there are general theorems on properties of quadratic forms applicable to variational problems involving functionals defined on classes of vector valued functions of m independent variables and with higher order derivatives. These theorems have as consequences further theorems on properties of systems of partial differential equations, and existence and differentiability theorems are established.

The purpose here is to set down an extension of the highly developed theory of conjugate points for simple integral problems in the calculus of variations to multiple integral problems. The extension is afforded by the theory established in [7]. Here the multiple integral problems in mind are those where the integrand involves at most first order partial derivatives of a real valued function of m real variables. The Jacobi equation of the second variation is then a linear second