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ON THE ZEROS OF THE SOLUTIONS OF THE DIFFERENTIAL EQUATION $y^{(n)}(z) + p(z)y(z) = 0$.

RINA HADASS

In this paper sufficient conditions for disconjugacy and for nonoscillation of the equation $y^{(n)}(z) + p(z)y(z) = 0$ are given. For $n = 2m$ a theorem ensuring that no solution of this equation has two zeros of multiplicity m is obtained. Here the invariance of the equation under linear transformations of z is used.

In [6] Nehari considered the equation

$$y^{(n)}(z) + p_{n-1}(z)y^{(n-1)}(z) + \cdots + p_0(z)y(z) = 0 , \tag{1}$$

where the analytic functions $p_i(z)$, $i = 0, \cdots, n-1$ are regular in a given domain D, and obtained a disconjugacy theorem for bounded convex domains and a nonoscillation theorem for the unit disk. Equation (1) is called *disconjugate* in a domain D, if no nontrivial solution of (1) has more than $(n-1)$ zeros in D. (The zeros are counted by their multiplicity). The equation is called nonoscillatory in D, if no nontrivial solution has an infinite number of zeros in D.

In this paper we obtained related results for a special case of (1); i.e., for the equation

$$y^{(n)}(z) + p(z)y(z) = 0 , \tag{2}$$

where the analytic function $p(z)$ is regular in the unit disk.

Section 1 deals with the invariance of equation (2), where $p(z)$ is analytic in a general domain, under the linear transformation

$$\zeta = \frac{az+b}{cz+d} , \qquad ad - bc \neq 0 , \tag{3}$$

(Theorem 1). The invariance of

$$y''(z) + p(z)y(z) = 0 \tag{4}$$

played an important role in Nehari's results on this second order equation [3; 5].

In § 2 we obtain sufficient conditions for disconjugacy and nonoscillation of equation (2) in the unit disk (Theorem 2 and Theorem 4 respectively). From Theorem 2 and the invariance of (2) under the linear transformations (3) we get a sufficient condition for the disconjugacy of (2) in non-Euclidean disks (Theorem 3).