PACIFIC JOURNAL OF MATHEMATICS
Vol. 32, No. 1, 1970

ON LEFT QF-3 RINGS

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In this paper the following results are proved:

(i) Three classes of left QF-3 rings are closed under taking left quotient rings respectively.

(ii) A subcategory of left modules having dominant dimensions $\geqq 2$ over a right perfect left QF-3 ring R is equivalent to a category of all left fRf-modules, where f is a suitable idempotent of R.

(iii) In case a left QF-3 ring is obtained as the endomorphism ring of a generator, dominant dimensions ($\geqq 2$) of modules are closely connected with the vanishing of Ext-functors.

(iv) Three classes of left and right QF-3 rings are identical in case of perfect rings.

Let R be an associative ring having an identity element 1 and denote by ${}_RR$ (resp. R_R) a left (resp. right) R-module R. To generalize the notion of QF-3 algebras [18] we shall make the following definitions:

(1) R is said to be left QF-3, if ${}_RR$ has a direct summand Re (e is an idempotent of R) which is a faithful, injective left ideal.

(2) R is said to be left QF-3^+, if the injective hull $E({}_RR)$ of ${}_RR$ is projective.

(3) R is said to be left QF-$3'$, if the injective hull $E({}_RR)$ of ${}_RR$ is torsionless in the sense of Bass [1].

Right QF-3, QF-3^+ and QF-$3'$ rings are defined in a similar fashion. It is obvious that the class of left QF-$3'$ rings is the most general class of the above three classes.

Our main purpose in this note is to introduce some generalizations of results for QF-3 algebras [11], [12], [15], [16], [17] and semi-primary QF-3 rings [4], [6], [13], [14] to the above generalized classes of rings.

We shall say that the dominant dimension of left (resp. right) R-module X, denoted by dom. dim ${}_RX$(resp. dom. dim X_R), is at least n, if there exists an injective resolution of X:

$$0 \longrightarrow X \longrightarrow W_1 \longrightarrow W_2 \longrightarrow \cdots \longrightarrow W_n$$

such that all W_i, $1 \leqq i \leqq n$, are torsionless. Then it is clear that R is left (resp. right) QF-$3'$ if and only if dom. dim ${}_RR$ (resp. dom. dim R_R) $\geqq$ 1.

In §1 we shall show that each class defined as above is closed under taking quotient rings (not necessarily classical), that is, a left