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EXTENDING HOMEOMORPHISMS

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Theorem 1 of this paper establishes a necessary and sufficient condition that a locally flat imbedding $f: B^k \to R^n$ of a k-cell in euclidean n-space R^n admits an extension to a homeomorphism $F: R^n \to R^n$ onto R^n such that $F \mid (R^n - B^k)$ is a diffeomorphism which is the identity outside some compact set in R^n. An analogous result for locally flat imbeddings of a euclidean $(n-1)$-sphere into R^n is proved. A lemma which generalizes a theorem of Huebsch and Morse concerning Schoenflies extensions without interior differential singularities is also established.

Let the points of euclidean n-space R^n be written $x = (x^1, \cdots, x^n)$, and provide R^n with the usual euclidean norm $\|x\| = [\Sigma(x^i)^2]^{1/2}$. We set $S_r = \{x \in R^n \mid \|x\| = r\}$, (and $S = S_1$). If M is a topological $(n-1)$-sphere in R^n, we denote the bounded component of $R^n - M$ by $\mathring{J}M$, and the closure of $\mathring{J}M$ in R^n by JM. We refer the reader to § 1 of [2] for the definition of the terms admissible cone K_z, conical point, axis of singular approach, and cone $K_z(\Sigma)$, where Σ is a euclidean $(n-1)$-sphere in R^n.

LEMMA 1. *Let z be an arbitrary point of S and φ a sense-preserving homeomorphism into R^n of an open neighborhood N of S such that φ carries points inside S to points inside $\varphi(S)$, and $\varphi \mid (N - S)$ is a C^m-diffeomorphism. There then exists a homeomorphism Φ of R^n onto R^n and a cone K_z* (resp. $\check{K}_z$) *with axis interiorly normal* (resp. *exteriorly normal*) *to S at z, such that if $X \subset N$ is a sufficiently small open neighborhood of S,*

$$\Phi(x) = \varphi(x) \qquad [x \in X - \{K_z(S) \cup \check{K}_z\}] \, ,$$

$\Phi \mid (R^n - S)$ is a C^m-diffeomorphism, and Φ is the identity outside some compact set in R^n.

REMARK. We note that a direct application of the proof of Theorem 1.2 of [2] will yield the conclusions of Lemma 1 except for single differential singularities in each component of $R^n - S$.

Proof of Lemma 1. The proof of Lemma 1 will be a variation of the proof of Theorem 1.2 of [2]. We can assume that $0 \in \mathring{J}\varphi(S)$. Let $\delta \in (\frac{1}{2}, 1)$ be a constant so near 1 that $S_\delta \subset N$. Using Theorem 1.1 of [2], there is a homeomorphism $f: JS \to R^n$ into R^n such that