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ON DISPERSIVE OPERATORS IN BANACH LATTICES

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Dispersive operators were introduced by R. S. Phillips for characterization of infinitesimal generators of nonnegative contraction semigroups in Banach lattices. Later other definitions of dispersiveness were given by M. Hasegawa and K. Sato. H. Kunita, for the purpose of application to Markov processes, introduced the notion of complete γ-dispersiveness which characterizes the infinitesimal generators of e-majoration preserving nonnegative semigroups T_t with norm $\leqq e^{\gamma t}$. In this paper we will give a unified treatment of these results. Further, we will clarify the relation between dispersiveness and dissipativeness in some cases. We consider also characterization of infinitesimal generators of nonnegative semigroups without norm conditions.

Let $\mathfrak{B}$ be a Banach lattice. That is, $\mathfrak{B}$ is a vector lattice and a real Banach space at the same time and $|f| \leqq |g|$ implies $\|f\| \leqq \|g\|$. We use the notations $f^+ = f \vee 0$, $f^- = -(f \wedge 0)$, and $|f| = f \vee (-f)$. Following Kunita [8], let $\tilde{\mathfrak{B}}$ be a vector lattice which is an extension of $\mathfrak{B}$, and let e be an element of $\tilde{\mathfrak{B}}$. We say that an operator T is *e-majoration preserving* if $f \leqq e$ implies $Tf \leqq e$. Let $\boldsymbol{G}$ be the set of infinitesimal generators of strongly continuous semigroups of linear operators in $\mathfrak{B}$. For real numbers $M \geqq 1$ and γ, let $\boldsymbol{G}(M, \gamma)$ be the set of $A \in \boldsymbol{G}$ such that the generated semigroup T_t satisfies $\|T_t\| \leqq Me^{\gamma t}$, $\boldsymbol{G}^e$ be the set of $A \in \boldsymbol{G}$ such that T_t is e-majoration preserving, and further, let $\boldsymbol{G}^e(M, \gamma) = \boldsymbol{G}(M, \gamma) \cap \boldsymbol{G}^e$. For linear operators, 0-majoration preserving is the same as nonnegativity and $\boldsymbol{G}^0$ is denoted by $\boldsymbol{G}^+$. We assume that e satisfies

$$f \in \mathfrak{B} \quad \text{implies} \quad f \wedge e \in \mathfrak{B}\,; \tag{0.1}$$

$$f \wedge \alpha e \text{ converges weakly to } f \wedge 0 \text{ as } \alpha \to 0+ \text{ for each } f \in \mathfrak{B}\,; \tag{0.2}$$

$$e \geqq 0\,. \tag{0.3}$$

Note that $f \wedge \alpha e \in \mathfrak{B}$ for $\alpha > 0$ by (0.1). We call a real-valued functional $\psi_e(f, g)$ on $\mathfrak{B} \times \mathfrak{B}$ *e-gauge functional*, if the following are satisfied:

$$\begin{gathered}\text{If } g \leqq e \text{ and } \alpha > 0 \text{ then } \psi_e(f, \alpha(f \wedge e - g)) \geqq 0 \text{ and} \\ \psi_e(f, \alpha(g - f \wedge e)) \leqq 0\,;\end{gathered} \tag{0.4}$$

$$\psi_e(f, g + h) \leqq \|g\| + \psi_e(f, h)\,; \tag{0.5}$$