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THEOREMS ON CESÀRO SUMMABILITY OF SERIES

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1.1. We consider the Cesàro summability, for integral orders, of the series

$$\sum_{\nu=0}^{\infty} a_\nu d_\nu \,. \tag{1.1}$$

In this paper we establish equivalence theorems for the series (1.1) which are valid for a substantial class of sequences d_ν including $e^{-\nu}$ and $\nu^{-\delta}$. Results of this character, but not overlapping with those in this paper, were given by Hardy and Littlewood and by Andersen. Andersen's result was extended by Bosanquet and Chow, and further extended by Bosanquet.

Notation. 1.2. We write $A_n^0 = A_n = a_0 + a_1 + \cdots + a_n$,

$$A_n^k = A_0^{k-1} + A_1^{k-1} + \cdots + A_n^{k-1}$$

and we get the identities: See Hardy [8].

$$A_n^k = \sum_{\nu=0}^{n} B_{n-\nu}^{k-1} A_\nu \,, \tag{1.2}$$

$$A_n^k = \sum_{\nu=0}^{n} B_{n-\nu}^{k} a_\nu \,, \tag{1.3}$$

where

$$B_{n-\nu}^k = \binom{n-\nu+k}{k} \,; \tag{1.4}$$

$E_n^k = A_n^k$ when $a_0 = 1$, $a_n = 0$, for $n > 0$, i.e., when $A_n = 1$, for all n.
Hence

$$E_n^k = \binom{n+k}{k} \sim \frac{n^k}{k!} \,. \tag{1.5}$$

If

$$\frac{A_n^k}{E_n^k} \to A, \text{ when } n \to \infty, \tag{1.6}$$

or equivalently if

$$\frac{k!A_n^k}{n^k} \to A, \text{ when } n \to \infty, \tag{1.7}$$

then we say that $\sum_{n=0}^{\infty} a_n$ is summable (C, k) to sum A and we write