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THE AMBIENT HOMEOMORPHY OF AN INCOMPLETE SUBSPACE OF INFINITE-DIMENSIONAL HILBERT SPACES

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The pair (H, H_f) is studied from a topological point of view (where H is an infinite-dimensional Hilbert space and H_f is the linear span in H of an orthonormal basis), and a complete characterization is obtained of the images of H_f under homeomorphisms of H onto itself. As the characterization is topological and essentially local in nature, it is applicable in the context of Hilbert manifolds and provides a characterization of (H, H_f)-manifold pairs (M, N) (with M an H-manifold and N an H_f-manifold lying in M so that each coordinate chart f of M may be taken to be a homeomorphism of pairs $(U, U \cap N) \xrightarrow{f} (f(U), f(U) \cap H_f))$.

This implies that in the countably infinite Cartesian product of H with itself, the infinite (weak) direct sum of H_f with itself is homeomorphic to H_f (the two form such a pair), and that if K is a locally finite-dimensional simplicial complex equipped with the barycentric metric (inducing the Euclidean metric on each simplex) and if no vertex-star of K contains more than $\dim(H)$ vertices, then $(K \times H, K \times H_f)$ is an (H, H_f)-manifold pair.

These results are used in [10] to study H_f-manifolds much more intensively to obtain results previously available only for H-manifolds or in the case that H_f is separable, i.e., connected H_f-manifolds are homeomorphic to open subsets of H_f, homotopy-equivalent H_f-manifolds are homeomorphic, and there is an essentially unique completion of an H_f-manifold into an H-manifold, yielding an (H, H_f)-pair.

It should be remarked that this characterization has already been achieved for separable Hilbert spaces by R. D. Anderson [1] and by C. Bessaga and A. Pełczynski [5], and that the observations concerning (H, H_f)-manifold pairs have been made by T. A. Chapman [6, 7] in that case. (Chapman then proceeded to obtain most of the results of [10] in the separable case by methods which seem at the moment to be limited to separability.)

Throughout the discussion, X will denote some complete metric space, and $\mathscr{H}(X)$, the group of all homeomorphisms of X onto itself. The term "isotopy" ("isotopic") will be understood as an abbreviation for "invertible, ambient isotopy", that is, a map $F: X \times [0, 1] \rightarrow X$ such that the function $G: X \times [0, 1] \rightarrow X \times [0, 1]$ defined from F by setting $G(x, t) = (F(x, t), t)$ is a homeomorphism. (When an embedding