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SUFFICIENT CONDITIONS FOR A RIEMANNIAN MANIFOLD TO BE LOCALLY SYMMETRIC

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In a locally symmetric Riemannian manifold the scalar curvature is constant and each k-th covariant derivative of the Riemannian curvature tensor vanishes. In this note, we show that if the covariant derivatives of the Riemannian curvature tensor satisfy some algebraic conditions at each point, then the Riemannian manifold is locally symmetric.

Let R be the Riemannian curvature tensor of a Riemannian manifold M^m with a positive-definite metric tensor g. Manifolds and tensors are assumed to be of class C^∞ unless otherwise stated. We denote by ∇ the Riemannian connection defined by g. For tangent vectors X and Y, we consider $R(X, Y)$ as a derivation of the tensor algebra at each point. A conjecture by K. Nomizu [4] is that $R(X, Y)\cdot R = 0$ on a complete and irreducible manifold $M^m (m \geqq 3)$ implies $\nabla R = 0$, that is, M^m is locally symmetric. Here we consider some additional conditions.

For an integer k and tangent vectors $V_k, \cdots, V_1$ at a point p of M^m, we adopt a notation:

$$\begin{aligned}(\nabla_V^k R) &= (V_k, V_{k-1}, \cdots, V_1; \nabla^k R)\\ &= (V_k^t V_{k-1}^s \cdots V_1^r \nabla_t \nabla_s \cdots \nabla_r R^a_{bcd}) ,\end{aligned}$$

where V_k^t, etc., are components of V_k, etc., and $\nabla_t \nabla_s \cdots \nabla_r R^a_{bcd}$ are components of the k-th covariant derivative $\nabla^k R$ of R in local coordinates.

PROPOSITION 1. *Let $M^m (m \geqq 3)$ be a real analytic Riemannian manifold. Assume that*
(1.0) *the restricted holonomy group is irreducible,*
(1.1) $R(X, Y)\cdot R = 0$,
(1.2) $R(X, Y)\cdot(\nabla_V^k R) = 0$ *for* $k = 1, 2, \cdots$.
Then M^m is locally symmetric.

Here we note that condition (1.0) means that it holds at some, hence every, point and condition (1.1), and (1.2), mean that for any point p and for any tangent vectors $X, Y, V_k, \cdots, V_1$ at p, they hold.

PROPOSITION 2. *Let $M^m (m \geqq 3)$ be a Riemannian manifold. Assume* (1.1) *and* (1.2) *and that*