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ON A CLASS OF TOPOLOGICAL ALGEBRAS

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This note introduces a class of topological algebras, called A-convex, which generalize the notion of locally m-convex algebras. They include a number of function space algebras which are not locally m-convex. Certain of these algebras admit a modified Gel'fand type representation in a space of vector-valued functions without invoking commutativity requirements. One seemingly obtains a new way of representing locally m-convex algebras. A-convex algebras are locally m-convex under the assumption of completeness of certain factor algebras in a suitable topology.

The definition of an A-convex algebra is given in § 2 together with some basic results. We define a condition, P-complete, such that every P-complete, A-convex algebra is locally m-convex. A class of important functions algebras whose seminorms are defined by certain types of weight functions is defined in § 3, see W. H. Summers [9]. Many of these are not locally m-convex, but are A-convex algebras. The definition and basic properties of an algebra of vector-valued functions where the index set is a completely regular Hausdorff space and the functions take values in (various) Banach algebras are given in § 4. Finally, the result is obtained in § 5 that each A-convex algebra is an inverse limit of A-normed (normed linear space with separately continuous multiplication) algebras. It is also shown that certain A-convex algebras can be represented as a subalgebra of an algebra of vector-valued functions. A sufficient condition for the representation to be valid is that A be barrelled. It is shown by means of an example that barrelled is not necessary for this representation to be valid.

Some of our results are analogous to various others given by P. D. Morris and D. E. Wulbert [7], G. R. Allan [1, 2], and R. M. Brooks [4, 5].

2. Basic definitions and results on multiplication. This paragraph is concerned with the introduction of some basic definitions and results on multiplications in a locally convex topological vector space. Let A be a locally convex topological vector space over the complex numbers K with a topology T determined either by a family N of absolutely convex neighborhoods of the origin or by a family P of