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EXPANSIVE AUTOMORPHISMS OF BANACH SPACES

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This paper treats two classes of invertible bounded linear operators on Banach spaces—expansive and uniformly expansive automorphisms—which include the hyperbolic automorphisms. Conditions for an automorphism to be expansive or uniformly expansive are given in terms of the location of its spectrum and approximate point spectrum with respect to the unit circle.

One of the tools used in [3] to determine all expansive automorphisms of compact connected Lie groups was the following result:

THEOREM 0. *Let T be an automorphism of a finite-dimensional real or complex normed linear space. Then a necessary and sufficient condition for T to be expansive is that $|\lambda| \neq 1$ for each complex characteristic root λ of T.*

Theorem 0 was deduced in [2] as a special case of a more general theorem, concerning topological vector spaces over arbitrary nondiscrete scalar fields, whose proof used algebraic methods leaning heavily on the assumption of finite dimensionality. In the present paper we use analytic considerations to treat the infinite-dimensional case.

The results we obtain were suggested by the following observation. In the finite-dimensional case, the condition for an automorphism to be expansive amounts to its being hyperbolic for some norm; in any Banach space, an automorphism is hyperbolic for some norm precisely when its spectrum is disjoint from the unit circle.

1. Preliminaries.

If B is a real or complex Banach space, we shall call any bounded linear operator on B having a bounded inverse on B an *automorphism* of B.

The most convenient definition of "expansive" for our purposes is the following.

DEFINITION 1. An automorphism T of a Banach space B is said to be *expansive* provided for each $x \in B$ with $||x|| = 1$ there exists some nonzero integer i such that $||T^i x|| \geqq 2$.

In this definition any norm equivalent to the given norm on B may be used, and the constant 2 may be replaced by any constant