PACIFIC JOURNAL OF MATHEMATICS
Vol. 35, No. 2, 1970

AN APPROXIMATION THEOREM FOR SUBALGEBRAS OF H∞

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Let E be a closed subset of the unitcircle $T = \{z : |z| = 1\}$ and denote by B_E the algebra of all functions which are bounded and continuous on the set $X = \{z : |z| \leqq 1 \ \& \ z \notin E\}$ and analytic in $D = \{z : |z| < 1\}$.

The main result of this paper (Theorem 1) is that there exist an open set V_E containing X such that every $f \in B_E$ can be approximated uniformly on X by functions being analytic in V_E.

The algebra B_E was introduced in [4] by E. A. Heard and J. H. Wells.

In [4] they characterize the interpolationsets for B_E. At the end of their paper they remark that the question of whether D is dense in the maximal ideal space $M(B_E)$ of B_E is open in case E is a proper nonempty subset of T. As a corollary of Theorem 1 we prove that D is dense in $M(B_E)$. (In proving the corollary we of course use the Carleson corona-theorem [1]).

This corollary has also been proved recently by Jaqueline Detraz in [3] where it follows from the very interesting fact that the restriction map from $M(H^\infty)$ (the maximal ideal space of $H^\infty(D)$) to $M(B_E)$ is onto. This is the main theorem of [3, Th. 2]. [3] contains also other results about B_E that we do not prove her. However, Theorem 2 of [3] can also be proved by using the main result of this paper together with the Carleson corona-theorem since Theorem 2 of [3] is equivalent with the fact that D is dense in $M(B_E)$. But the proof of Theorem 2 in [3] is more direct and do not involve the Carleson corona theorem.

Through the whole paper $r_0 > 1$ will be a fixed real number.

Define an open set V_E by $V_E = X \cup \{z : 1 \leqq |z| < r_0 \ \& \ \frac{z}{|z|} \notin V\}$

THEOREM 1. *For every $f \in B_E$ and every $\varepsilon > 0$ there exist a function g analytic in V_E such that $\|f - g\|_X < \varepsilon$.*

LEMMA 1. *Suppose $f \in B_E$ and e is a continuously differensiable function on T with compact support contained in $\mathbf{C} \backslash E$.*

If $f = u + iv$ and we define $u_1(\theta) = u(\theta)e(\theta)$ $(\theta \in (-\pi, \pi])$, then the function

$$f_1(z) = \frac{1}{2\pi} \int_{\pi}^{\pi} \frac{e^{i\theta} + z}{e^{i\theta} - z} u_1(\theta) d\theta = u_1(z) + v_1(z)$$