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NOTES ON COMMUTATIVE POWER JOINED SEMIGROUPS

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Let S be a commutative semigroup. The main theorem in this paper is to prove that the following two conditions are equivalent: (1) For all $a, b \in S$ there are positive integers m, n such that $a^m = b^n$. (2) For all $a, b \in S$, $a^l = a^m b^n$, $b^r = b^s a^t$ for some l, m, n, r, s, t. As a consequence of the theorem, the authors prove that a commutative archimedean semigroup S without idempotent is power joined if and only if the structure group of S is a torsion group.

Let S be a commutative archimedean semigroup without idempotent. Consider the following question: "Under what condition on the structure group (defined below) of S will S be power joined?" Levin proved in [4] that if S is finitely generated, equivalently if the structure group of S is finite, then S is power joined. Also he obtained a necessary and sufficient condition for S to be power joined. The following is Theorem 2 in [4]:

THEOREM 1. *Let S be a commutative, archimedean semigroup without idempotent. Let $G_a = S/\rho_a$ be the structure group of S determined by a. Then S is power joined if and only if G_a is periodic and the congruence class containing a modulo ρ_a is power joined.*

If we assume that S is additionally cancellative, that is, S is an $\mathfrak{N}$-semigroup, then the answer is simple. The following is due to Chrislock [1, 2].

THEOREM 2. *An $\mathfrak{N}$-semigroup S is power joined if and only if G_a is periodic for some $a \in S$, equivalently for all $a \in S$.*

Naturally the following question is raised: Can Theorem 1 be improved such that Theorem 2 is extended to S in Theorem 1? The question is affirmative. In this paper we study the problem for more general case, i.e., for commutative archimedean semigroups. The main theorem of this paper asserts that a commutative semigroup S is power joined if and only if it is archimedean and its group homomorphic images are periodic. As a corollary we can answer the above question.

Semigroups are assumed to be commutative throughout this paper.