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VOLTERRA TRANSFORMATIONS OF THE WIENER MEASURE ON THE SPACE OF CONTINUOUS FUNCTIONS OF TWO VARIABLES

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The transformation of Wiener integrals over the space C_2 of continuous functions of two variables by a Volterra operator T is investigated. The operator T is defined for functions $x \in C_2$ by

$$Tx(s, t) = x(s, t) + \int_0^s \int_0^t K(u, v)x(u, v)dudv ,$$

where the kernel $K(u, v)$ is continuous. A stochastic integral analogous to K. Ito's is defined and used to determine a Jacobian $J(x)$ for T such that if $F(x)$ is a Wiener measurable functional, Γ a Wiener measurable set, and m Wiener measure, $\int_\Gamma F(x)dm = \int_{T^{-1}(\Gamma)} F(Tx)J(x)dm.$

Let C_2 be the collection of real valued functions f defined on $D = [0, 1] \times [0, 1]$ such that $f(0, t) = f(s, 0) = 0$. The space C_2 is topologized by the sup-norm. In [3], Yeh defined a measure m on C_2 over the Borel σ-algebra and extended it to the Caratheodory σ-algebra relative to m. It is the purpose of this paper to investigate the transformation of the measure m when the elements of C_2 are transformed by a Volterra integral operator of the second kind. The effect of such transformations in the Wiener space of continuous functions of one variable was studied by Cameron and Martin in [1].

Let $0 = s_0 < s_1 < \cdots < s_m \leqq 1$ and $0 = t_0 < t_1 < \cdots < t_n \leqq 1$ and let E be a nm-dimensional Borel set. We denote by $\mathfrak{F}(s_1, \cdots, s_m, t_1, \cdots, t_n)$ the σ-algebra of sets of the form $\{x \in C_2: [x(s_1, t_1), \cdots, x(s_m, t_n)] \in E\}$ and let $\mathfrak{F}_0 = \cup \mathfrak{F}(s_1, \cdots, s_m, t_1, \cdots, t_n)$ where the union is over all such partitions of D. The measure m is given on $\mathfrak{F}(s_1, \cdots, s_m, t_1, \cdots, t_n)$ by

$$\begin{aligned} &m\{x \in C_2: [x(s_1, t_1), \cdots, x(s_m, t_n)] \in E\} \\ (1.1) \qquad &= K(s_1, \cdots, s_m, t_1, \cdots, t_n) \\ &\cdot \int_E (mn) \int W(s_1, \cdots, s_m, t_1, \cdots, t_n, u_{11}, \cdots, u_{mn})du_{11}, \cdots, du_{mn} , \end{aligned}$$

where

$$\begin{aligned} &K(s_1, \cdots, s_m, t_1, \cdots, t_n) \\ &\quad = \{(2\pi)^{-mn}[s_1(s_2 - s_1) \cdots (s_m - s_{m-1})]^n[t_1(t_2 - t_1) \cdots (t_n - t_{n-1})]^m\}^{\frac{1}{2}} , \end{aligned}$$