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CHARACTERIZATION OF THE ENDOMORPHISM RINGS OF DIVISIBLE TORSION MODULES AND REDUCED COMPLETE TORSION-FREE MODULES OVER COMPLETE DISCRETE VALUATION RINGS

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Let R be a complete discrete valuation ring (possibly non-commutative). If K is the quotient field of R then there is an isomorphism between the category of divisible torsion R-modules G and the category of reduced complete torsion-free R-modules H given by $G \to H = \mathbf{Hom}_R(K/R, G)$. Moreover, the R-endomorphism ring $E(G)$ is naturally isomorphic to the R-endomorphism ring $E(H)$ of H. It is the purpose of this paper to find necessary and sufficient conditions for an abstract ring to be isomorphic to the R-endomorphism ring of such an R-module.

Our problem has already been solved in the special case where R is a (not necessarily commutative) field. In [16] Wolfson characterized the ring E of all linear transformations of a vector space over a field by the following four properties: (1) E_0, the socle of E, is not a zero ring, and is contained in every nonzero two-sided ideal of E. (2) If L is a left ideal of E which is annihilated on the right only by zero, then $E_0 \subset L$. (3) The sum of two left (right) annihilators is a left (right) annihilator. (4) E possesses an identity element. Our main theorem may be considered as an extension of Wolfson's beautiful result to the case of an arbitrary complete discrete valuation ring. So, for example, in passing from the vector space case to this general one, "subspace" now becomes "direct summand" and "zero ideal" translates to "Jacobson radical". If H is a vector space over a field R, then the structure of its R-endomorphism ring $E(H)$ is to a large extent determined by the ideal $E_0(H)$ of all R-endomorphisms which map H onto a subspace of finite rank. In this case $E_0(H)$ is the socle of $E(H)$, the sum of all minimal left (right) ideals of $E(H)$. If R is an arbitrary complete discrete valuation ring and H a reduced complete torsion-free R-module, then $E_0(H)$ determines again the behavior of the entire ring $E(H)$. The proper generalization now reads: $E_0(H)$ is the sum of all minimal nonradical left (right) ideals of $E(H)$. Here we call an ideal of a ring E *nonradical* if it is not contained in the Jacobson radical $J(E)$ of E. And by a minimal nonradical ideal we mean an ideal I which is nonradical and has the property that every ideal of E which is properly contained in I belongs to $J(E)$.