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AN EXTENSION OF SOME RESULTS OF TAKESAKI IN THE REDUCTION THEORY OF VON NEUMANN ALGEBRAS

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Briefly, the results in this paper are that both for measurable fields of von Neumann algebras and for families of measurable fields of operators, pointwise isomorphism implies isomorphism.

In the special case when half the measurable fields considered are constant, these results were established by Takesaki. If the Borel space on which the fields are defined is standard, the results can be established by classical means; in the case considered by Takesaki they are due to von Neumann.

For the results of the present paper, two new tools seem to be needed. The first is a measurable choice theorem of Aumann which generalizes the classical one. This has already been applied to reduction theory by Flensted-Jensen. The second is a criterion for a von Neumann algebra containing the diagonal operators to be decomposable: it should consist of decomposable operators. This answers a question of Dixmier.

We shall use the terminology of reduction theory developed in [2], Chapitre II.

2. LEMMA (Aumann). *Let T be a Borel space and let X be a standard Borel space. Let G be a Borel subset of $T \times Y$ such that the projection of G onto T is all of T. Let there be given a finite measure on T. Then there exists a measurable map g: $T \rightarrow X$ such that $(t, g(t)) \in G$ for almost all $t \in T$.*

Proof. See [1]. The proof is by reduction to the case that T is standard.

3. THEOREM. *Let T be a Borel space, and suppose given a finite measure on T and a measurable field of Hilbert spaces on T with direct integral H. Let A and B be decomposable von Neumann algebras in H. If for each $t \in T$ there is a spatial isomorphism of $A(t)$ onto $B(t)$ then there exists a decomposable spatial isomorphism of A onto B. This statement also holds with the word "spatial" removed.*