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SPECTRAL THEORY FOR A FIRST-ORDER SYMMETRIC SYSTEM OF ORDINARY DIFFERENTIAL OPERATORS

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For a symmetric differential expression associated with a first order system

$$A_0(t)x' + A(t)x, \quad a < t < b$$

where A_0 and A are $n \times n$ matrices and x is an $n \times 1$ vector, a spectral decomposition will be developed. That is, if S is a closed symmetric differential operator determined by the differential system, the explicit nature of the generalized resolutions of the identity for all the self-adjoint extensions of S in any Hilbert space will be determined in terms of a fundamental matrix and spectral matrices associated with these extensions. An important aspect is that these self-adjoint extensions may be defined in Hilbert spaces larger than the natural one $\mathscr{H}$ in which the operator S is defined.

The development proceeds as in Coddington [5]; however, the consideration of systems of differential equations introduces matrix techniques and notation. It is hoped that this formulation will have application to such problems as open end (infinite time) control theory problems, and facilitate the canonical formulation of the associated spectral analysis.

Preliminary definitions. Let $\mathscr{H}$ be a Hilbert space with an inner product $(\ ,\ )$.

(1) *Generalized Resolution of the Identity.* Let $F = \{F(\lambda)\}$ be a family of bounded self-adjoint operators in $\mathscr{H}$, depending on real λ, such that:

(i) $F(\lambda) \geqq F(\mu)$, $\lambda > \mu$,

(ii) $F(\lambda + 0) = F(\lambda)$,

(iii) $F(\lambda) \to I$, as $\lambda \to +\infty$,
$F(\lambda) \to 0$, as $\lambda \to -\infty$,

then F is a generalized resolution of the identity.

The family F is said to be associated with a symmetric operator Z (or F is a "spectral function" for Z, Naimark [7]) if

$$(Zu, v) = \int \lambda d(F(\lambda)u, v),$$

and