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ON FREDHOLM TRANSFORMATIONS IN YEH-WIENER SPACE

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Let C_Y denote the Yeh-Wiener space, i.e., the space of all real-valued continuous functions $f(x, y)$ on $I^2 \equiv [0, 1] \times [0, 1]$ such that $f(0, y) = f(x, 0) \equiv 0$. Yeh has defined a Gaussian probability measure on C_Y such that the mean of the process

$$m(x, y) \equiv \int_{C_Y} f(x, y) d_Y f = 0$$

and the convariance

$$R(s, t, x, y) \equiv \int_{C_Y} f(s, t) f(x, y) d_Y f = (1/2) \min(s, x) \min(t, y) .$$

Consider now a linear transformation of C_Y onto C_Y of the form

$$\begin{aligned} T: f(x, y) &\to g(x, y) \\ &= f(x, y) + \int_{I^2} K(x, y, s, t) f(s, t) ds dt , \end{aligned} \tag{1.1}$$

which is often called a Fredholm transformation. The main purpose of this paper is to find the corresponding Radon-Nikodym derivative thus showing how the Yeh-Wiener integrals transform under the transformation.

The transformations considered here contain the Volterra transformation

$$T_1[f(x, y)] = f(x, y) + \int_0^y \int_0^x K(x, y, s, t) f(s, t) ds dt$$

as a special case.

Such transformations in Wiener space have been studied a great deal by Cameron and Martin [1], Woodward [9], Segal [5], [6], and Shepp [7], and the results have proved very useful in the evaluation of various Wiener integrals.

The transformation theorems in this paper are based on stochastic integrals called the generalized Paley-Wiener-Zygmud (P.W.Z.) integrals given in [3] and [4]. For a function $h(x, y) \in L^2(I^2)$ and $f(x, y) \in C_Y$, the generalized P.W.Z. integral is defined to be

$$\int_{I^2} h f d^* f \equiv \lim_{n \to \infty} \int_{I^2} (hf)_n df , \tag{1.2}$$