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ON THE ENDOMORPHISM RING OF AN ABELIAN p-GROUP, AND OF A LARGE SUBGROUP

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For an abelian p-group G, denote the endomorphism ring of G by $E(G)$, the ideal of small endomorphisms by $E_s(G)$ and the quotient ring $E(G)/E_s(G)$ by $S(G)$. It is not difficult to show that for a large subgroup L of G, the map that sends an endomorphism of G to its restriction on L induces a monomorphism $S(G) \rightarrow S(L)$. We show that if B_1 is a large subgroup of a group B_2 which is a direct sum of cyclic p-groups and is of cardinality not more than $2^{\aleph_0}$ and R_1 and R_2 are suitable subgroups of $E(B_1)$ and $E(B_2)$, then there are groups G_1 and G_2 having B_1 and B_2 as basic subgroups such that G_1 is large in G_2 and $S(G_i) \cong R_i/(E_s(B_i) \cap R_i)$, $(i = 1, 2)$.

As with Corner's result [2, Th. 2.1], much of the value of this theorem is in producing examples. Although it is clear that these theorems can be used to exhibit counterexamples to several obvious conjectures, we will confine ourselves to giving a counterexample to a theorem of Paul Hill which asserts that the endomorphism ring of a p-group G (with $p \neq 2$) is generated by its units if this property holds for any large subgroup of G. To show that our example has the necessary properties we give, in § 3, some conditions equivalent to a ring of endomorphisms being generated by its units.

All groups considered will be abelian. For a p-group G, we will denote the set of automorphisms of G by $A(G)$ and the subring of $E(G)$ that $A(G)$ generates by $\{A(G)\}$. The ideal of endomorphisms that annihilate the socle of G will be denoted by $N(G)$. As we will be writing morphisms on the right, the restriction of an endomorphism α of a group G to a fully invariant subgroup L of G will be written as $(L \mid \alpha)$. For information on large subgroups and small homomorphisms the reader is referred to Pierce's paper [6]. The topology on a group or its endomorphism ring will always be the p-adic topology. The closure of a set X will be denoted by X^-.

2. An extension of Corner's Theorem.

Given a torsion complete p-group C with unbounded basic subgroup B of cardinality not more than $2^{\aleph_0}$, Corner's theorem [2, Th. 2.1] gives sufficient conditions on a subring R of $E(B)$ for there to exist a pure subgroup G of C containing B such that $E(G) = E_s(G) \oplus R$. It is necessary for our example in section 4 to weaken slightly the conditions on the ring R. In doing this we obtain a group G such that $E(G) = E_s(G) + R$ without neces-