PACIFIC JOURNAL OF MATHEMATICS
Vol. 41, No. 1, 1972

GROUP REPRESENTATIONS AND THE ADAMS SPECTRAL SEQUENCE

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Homotopy groups admit primary operations analogous to the Steenrod operations in ordinary cohomology theory and a good understanding of them seems vital to interpreting patterns in the homotopy of spheres.

Also, it has been known for a long time that a type of Steenrod algebra acts in $\mathrm{Ext}_A(Z_p, Z_p)$ if A is a cocommutative Hopf algebra. Recently, D. S. Kahn showed that in the E_2 term of the Adams spectral sequence $\mathrm{Ext}^{**}_{\mathscr{A}(2)}(Z_2, Z_2)$, certain of these operations on infinite cycles converge to the graded elements associated to the actual homotopy operations. Also, on infinite cycles, he showed how this structure determined some differentials.

In this paper, we further explore the relations between the operations in $\mathrm{Ext}^{**}_{\mathscr{A}(p)}(Z_p, Z_p)$ and differentials in the Adams spectral sequence. In particular, for elements which need not be infinite cycles, we prove

THEOREM 4.1.1. **(a) There are operations Sq^i in $\mathrm{Ext}_{\mathscr{A}(2)}(Z_2, Z_2)$ so that**

$$\partial_2(Sq^i(a)) = \begin{cases} h_0 Sq^{i+1}(a), & i \equiv s(2) \\ 0 & \text{otherwise} \end{cases},$$

for $a \in \mathrm{Ext}^{r,s}_{\mathscr{A}(2)}(Z_2, Z_2)$.

(b) There are operations $\mathscr{P}^i$, $\beta\mathscr{P}^i$ in $\mathrm{Ext}_{\mathscr{A}(p)}(Z_p, Z_p)$ for p an odd prime so that

$$\partial_2(\mathscr{P}^i(a)) = \alpha_0 \beta\mathscr{P}^i(a),$$

for $a \in \mathrm{Ext}^{r,s}_{\mathscr{A}(p)}(Z_p, Z_p)$. (Here, Sq^i takes $\mathrm{Ext}^{s,r}$ homomorphically to $\mathrm{Ext}^{s+i,2r}$ while $\mathscr{P}^i$ takes $\mathrm{Ext}^{s,r}$ to $\mathrm{Ext}^{s+(2i-r)(p-1),pr}$, and $\beta\mathscr{P}^i(\mathrm{Ext}^{s,r}) \subset \mathrm{Ext}^{s+(2i-r)(p-1)+1,pr}$.)

These operations are readily computable in the Ext groups. (Methods for calculating them are given in §6 and [2], [19].) For example, $Sq^0(h_i) = h_{i+1}$, $Sq^1(h_i) = h_i^2$ where h_i is the nonzero element in $\mathrm{Ext}^{1,2^i}$ dual to Sq^{2^i}. (Our notation for elements in Ext is that of [13].) Applying 4.1.1, $\partial_2(h_i) = h_0 h_{i-1}^2$, which is nonzero for i greater than 3. Consequently, we recover

COROLLARY (Adams). *An element of Hopf invariant one* (mod 2) *exists in $\pi_{n+s}(S^n)$ if and only if $n > s$ and $s = 1, 3, 7$.*

(The Hopf invariant of a homotopy class α is nonzero in Z_p-cohomology if and only if α is represented by an infinite cycle in