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ABSOLUTE TOTAL-EFFECTIVE $(N, p_n)(C, 1)$ METHOD

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In the direction of the total-effectiveness of a $(N, p_n)(C, 1)$ method, results concerning the summability of a Lebesgue Fourier series and its conjugate series by such a method are known. Supporting the observation that generally bounded variation is the property associated with absolute summability in the same way in which continuity is associated with ordinary summability, the absolute total-effectiveness of a $(N, p_n)(C, 1)$ method is established in the present paper and the corresponding effectiveness of the (C) method is deduced as a particular case.

Throughout the present paper we use the definitions and notations of [7] without further explanation. The following additional notations for the conditions concerning $\{p_n\}$ are also used.

(1.1) $\{p_n\} \in RS$ means: $p_0 > 0$, $p_n \geqq 0$ $(n \geqq 1)$, $\{R_n\} \in BV$ and $\{S_n\} \in B$;

(1.2) $\{p_n\} \in MS$ means: $p_n > 0$, $p_{n+1}/p_n \leqq p_{n+2}/p_{n+1} \leqq 1$ $(n \geqq 0)$ and $\{S_n\} \in B$;

(1.3) $\{p_n\} \in NS$ means: $p_0 > 0$, $p_n \geqq 0$ $(n \geqq 1)$, $\{R_n\} \in B$, $\{S_n\} \in B$, $\{p_n\}$ and $\{\Delta p_n\}$ monotone .

As we shall see in section 5 of the present paper, $MS \subset NS$, but no interrelation is known between the sets of conditions RS and MS or NS.

Using a result due to Mears [15], Kwee [13] has proved that the following conditions:

$$p_n = o(|P_n|),\ n \rightarrow \infty \quad \text{and} \quad \sum_{n=1}^{\infty} \left| \frac{P_n}{P_{n+\nu}} - \frac{P_{n-1}}{P_{n+\nu-1}} \right| < \infty \ , \tag{1.4}$$

for all $\nu \geqq 1$, are necessary and sufficient for the absolute regularity of the (N, p_n) method. It may be observed that Lemma 1 and Lemma 2 of the present paper imply *a fortiori* that the (N, p_n) method is absolutely regular, under each of the conditions: $\{p_n\} \in RS$, $\{p_n\} \in NS$ and $\{p_n\} \in MS$.

Concerning the absolute Fourier-effectiveness, we have the following.

THEOREM A. *If $\{p_n\} \in RS$, then the (N, p_n) method is absolute Fourier-effective.*