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PEAK INTERPOLATION SETS FOR SOME ALGEBRAS OF ANALYTIC FUNCTIONS

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For certain algebras of analytic functions on holomorphically convex sets in C^n metric sufficient conditions are given for a set (not necessarily compact) to be an interpolation set. The results extend the Rudin-Carleson theorem for the disc algebra.

Let K be a compact subset of C^n which is holomorphically convex, i.e. K is the intersection of a decreasing sequence of pseudoconvex domains (see [4], Ch. 2). We denote by $H(K)$ the uniform closure on K of the algebra of all functions analytic in a neighborhood of K, and by $A(K)$ the algebra of all continuous functions on K analytic on K^0 (the interior of K). If E is any subset of the boundary ∂K of K then we denote by H_E^∞ the algebra of all bounded continuous functions on $K^0 \cup E$ which are analytic on K^0. We show that if the boundary of K is well behaved at each point of E, and E satisfies a metric condition which says roughly that E has zero 2-dimensional measure in the directions of the complex tangent and zero one dimensional measure in the orthogonal direction, then E is a peak interpolation set (in an appropriate sense) for $H^\infty_{E \cup (\partial K \setminus \bar{E})}$. If E is compact then it is a peak interpolation set in the usual sense ([2], p. 59) for the uniform algebra $H(K)$. We show also that if E has zero one-dimensional measure then the conditions on ∂K can be relaxed.

We say that ∂K is strictly pseudoconvex in a neighborhood of a point $\zeta \in \partial K$ if there is an open neighborhood V of ζ such that $V \cap \partial K$ is a C^2-submanifold of V and the Levi form is positive definite at ζ. Then we can find an open neighborhood V of ζ and a C^2 strictly plurisubharmonic function ρ in V such that $K \cap V = \{z \in V: \rho(z) \leqq 0\}$ and grad $\rho \neq 0$ on $V \cap \partial K$. (See [3] Prop. IX. A4).

LEMMA 1. *Let K be a holomorphically convex compact set in C^n and let ζ be a point of ∂K in a neighborhood of which ∂K is strictly pseudoconvex. We can find positive numbers m_ζ and M_ζ and $G_\zeta \in H(K)$, such that*

(a) $\operatorname{Re} G_\zeta(z) \geqq m_\zeta |\zeta - z|^2, z \in K$

(b) $\operatorname{Re} G_\zeta(z) \leqq M_\zeta |\zeta - z|^2, z \in \partial K$

(c) $\operatorname{grad} (\operatorname{Re} G_\zeta)(\zeta) = - \operatorname{grad} \rho(\zeta)$.

Proof. Put