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THE INFLATION—RESTRICTION THEOREM FOR AMITSUR COHOMOLOGY

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In this paper we develop a generalization of the classical exactness of the inflation—restriction sequence in group cohomology. Our main theorems relate the Amitsur cohomology of algebras to that of subalgebras.

1. Introduction. Throughout, R is a commutative ring, unadorned $\otimes$ means tensor product over R, all algebras are commutative, and if S is an R-algebra, S^j denotes the tensor product $S \otimes \cdots \otimes S$, j times. R-Alg and Ab denote the categories of commutative R-algebras and abelian groups, respectively.

For any R-algebra S there are R-algebra maps $\varepsilon_i^n \colon S^n \to S^{n+1}$ given by $\varepsilon_i^n(s_0 \otimes \cdots \otimes s_{n-1}) = s_0 \otimes \cdots \otimes s_{i-1} \otimes 1 \otimes s_i \otimes \cdots \otimes s_{n-1}$, $i = 0, 1, \cdots, n+1$. These are called the (n-dimensional) *co-face maps for S/R*. Generally the superscript will be suppressed. The co-face maps are easily seen to satisfy the co-face relations:

$$\varepsilon_i \varepsilon_j = \varepsilon_{j+1} \varepsilon_i \quad \text{for} \quad i \leqq j \, .$$

If $F\colon R\text{-Alg} \to Ab$ is any functor, the Amitsur cochain complex, $C(S/R, F)$, is defined by $C^n(S/R, F) = F(S^{n+1})$, $n = 0, 1, 2, \cdots$ [1, 2, 6]. The coboundary operator $d^n\colon F(S^{n+1}) \to F(S^{n+2})$ is given by $d^n = \sum_{i=0}^{n+1}(-1)^i F(\varepsilon_i)$. It is a consequence of the co-face relations that a complex results, i.e., that $d^{n+1}d^n = 0$. The homology $\operatorname{Ker} d^n/\operatorname{Im} d^{n-1}$ of this complex is the *Amitsur cohomology of S/R with coefficients in F*, denoted $H^n(S/R, F)$. As usual, $H^0(S/R, F) = \operatorname{Ker} d^0$.

Let $F_1\colon R\text{-Alg} \to Ab$ be another functor and let $\eta\colon F \to F_1$ be a natural transformation. Then $C(1, \eta) = \eta_{S^{n+1}}\colon F(S^{n+1}) \to F_1(S^{n+1})$ is a map of complexes and so induces a map $H^n(1, \eta)\colon H^n(S/R, F) \to H^n(S/R, F_1)$.

We say a sequence $0 \to F^{\omega} F_1 \chi F_2 \to 0$ is exact if $0 \to F(A) \xrightarrow{\omega_A} F_1(A) \xrightarrow{\chi_A} F_2(A) \to 0$ is an exact sequence of abelian groups for each R-algebra A. Indeed the usual long sequence results from a short exact sequence of coefficients.

THEOREM 1.1. [6, p. 47]. *Let $0 \to F \xrightarrow{\omega} F_1 \xrightarrow{\chi} F_2 \to 0$ be an exact sequence of functors. Then there are maps $\delta^n(S)$ making*

$$\cdots \longrightarrow H^{n-1}(S/R, F_2) \xrightarrow{\delta^{n-1}(S)} H^n(S/R, F) \xrightarrow{H^n(1,\omega)} H^n(S/R, F_1)$$
$$\xrightarrow{H^n(1,\chi)} H^n(S/R, F_2) \xrightarrow{\delta^n(S)} H^{n+1}(S/R, F) \longrightarrow \cdots$$