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TENSOR PRODUCTS OF PARTIALLY ORDERED GROUPS

JORGE MARTINEZ

All groups considered in this paper are abelian. It is concerned for the most part with defining suitable tensor products on categories of partially ordered groups. There is introduced the purely auxiliary notion of a partial vector space for the purpose of leading to a reasonable construction of a "vector lattice cover". The so-called o-tensor product from the category of p.o. groups into the category of lattice-ordered groups (l-groups) yields some surprising and surely disappointing results, such as that the functor $G \otimes_o (.)$ preserves monics if and only if G is trivially ordered. This follows from the fact that if G is trivially ordered then $G \otimes_o H$ is independent of the order on H and in fact l-isomorphic to the free l-group on the ordinary tensor product $G \otimes H$. It should be observed that the latter applies to torsion free groups only.

Section 2 is devoted to a discussion of vector lattice coverings. We give a categorical "construction" of such a cover, and establish the connection with the o-tensor product. More specifically, if the l-group G is free over some p.o. group H then the cover of G is obtained by o-tensoring the reals with H. Once again, the results or rather the lack of them, is surprising and perhaps revealing.

Finally we define the l-tensor product of l-groups, which leads us to the vector lattice cover via an alternate route.

The basic material on partially ordered algebraic structure may be found in [2] or [8]. For a discussion of free l-groups we refer the reader to [11], [12], [1], [6] and [7], listed more or less chronologically. Viswanathan has given a construction of a tensor product of partially ordered modules in [10] which reduces to the o-tensor product when restricted to groups. For the categorical background necessary in this context we suggest [9].

NOTATION. Z, Q and R will denote the totally ordered groups of integers, rationals and reals respectively, with the usual orderings. If $\{G_i \mid i \in I\}$ is an arbitrary family of l-groups, the cardinal sum, denoted by $\boxplus \{G_i \mid i \in I\}$ is the restricted direct sum with coordinatewise ordering. If A and B are subsets of a set X, proper containment of A in B is denoted by $A \subset B$. If $C \subseteqq X$ then $A \backslash C$ is the complement of C in A.