PACIFIC JOURNAL OF MATHEMATICS
Vol. 41, No. 3, 1972

PARTIAL ALGEBRAIC STRUCTURES ASSOCIATED WITH ORTHOMODULAR POSETS

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In this paper there are three main results:
I. An orthomodular poset with property C is essentially the same as an associative partial Boolean algebra.
II. If P is an orthomodular poset, then $S(P)$, the set of residuated maps on P, can be made into a weak partial Baer*-semigroup is such a way that P is isomorphic to the orthomodular poset of closed projections in $S(P)$.
III. If (P, M) is a conditional quantum logic, then the collection of all finite compositions of primitive operations (satisfying certain technical conditions) is a partial Baer*-semigroup.

It is assumed that the reader is familiar with the rudiments of the theory of orthomodular posets.

1. Introduction. Although partial algebraic structures seem to arise in many areas of mathematics, their systematic study seems to have been neglected. Among the few studies of such structures known to the author are the connection between congruence relations and partial algebras [4, 5], the coordinatization of partially ordered sets and partial Baer*-semigroups [6, 18], and considerations of the "logic" of quantum mechanics [9, 11, 12]. It is the purpose of this paper to consider some partial algebraic structures and to develop at least a small portion of their theory with the hope of stimulating further interest and research in this field.

By a partial algebraic structure we mean a set S with certain "partial" operations in the sense that these operations are defined only for certain elements of S. For example let S be an ordinary algebraic structure and let $T \subseteq S$. If we restrict the operations of S to those elements which are in T after the operations are perfomed then we obtain a partial algebraic structure. In particular, let $(S, \cdot)$ be a semigroup and let T be a nonempty subset of S. Define a relation $R \subseteq S \times S$ by $(a, b) \in R$ if $a \cdot b \in T$. We then have a map $R \rightarrow S$ given by $(a, b) \rightarrow a \cdot b$ and we call $(S, R, \cdot)$ a partial semigroup (also $(T, R \cap (T \times T), \cdot)$ is a partial semigroup). We say that these partial semigroups are *generated* by the subset T of the semigroup S. Notice that we have a partial associative law in these partial semigroups in the sense that if $(a, b) \in R$ and $(b, c) \in R$ then $(a \cdot b, c) \in R$ if and only if $(a, b \cdot c) \in R$ and in that case $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

As another example, let S be the set of all rectangular matrices with real entries. Then the natural addition can only be applied to