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LIE STRUCTURE OF PRIME RINGS OF CHARACTERISTIC 2

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In this paper the Lie structure of prime rings of characteristic 2 is discussed. Results on Lie ideals are obtained. These results are then applied to the group of units of the ring, and also to Lie ideals of the symmetric elements when the ring has an involution. This work extends recent results of I. N. Herstein, C. Lanski and T. S. Erickson on prime rings whose characteristic is not 2, and results of S. Montgomery on simple rings of characteristic 2.

1. Prime rings. We first extend the results of Herstein [5]. Unless otherwise specified, all rings will be associative. If R is a ring, R has a Lie structure given by the product $[x, y] = xy - yx$, for $x, y \in R$. A Lie ideal of R is any additive subgroup U of R with $[u, r] \in U$ for all $u \in U$ and $r \in R$. By a commutative Lie ideal we mean a Lie ideal which generates a commutative subring of R.

Denote the center of R by Z. We recall that if R is prime, then the nonzero elements of Z are not zero divisors in R. In this case, if $Z \neq 0$ and F is the quotient field of R, then $R \otimes_Z F$ is a prime ring, every element of which can be written in the form $r \otimes a^{-1}$ for $a \in Z$, $a \neq 0$. Thus $R \otimes_Z F$ is naturally isomorphic to RZ^{-1}, the localization of R at Z. We will consider R imbedded in RZ^{-1} in the usual way (see [2]).

We begin with some easy lemmas.

LEMMA 1. *If R is semi prime and U is a Lie ideal of R with $u^2 = 0$ for all $u \in U$, then $U = 0$.*

Proof. Let $u \in U$, $x \in R$ then $0 = (ux - xu)^2 = uxux - ux^2u + xuxu$. Right multiply by ux to obtain $(ux)^3 = 0$. Thus uR is a nil right ideal of index 3. Since R is semi prime, by Levitzki's Theorem [6; Lemma 1.1] $u = 0$.

LEMMA 2. *Suppose $2R = 0$ and U is a commutative Lie ideal of R. Then $u^2 \in Z$ for all $u \in U$.*

Proof. Let $u \in U$, $x \in R$. Then $ux + xu \in U$ so $uxu + u^2x = xu^2 + uxu$. Hence $u^2 \in Z$.

LEMMA 3. *Let R be prime and I a nonzero ideal of R. If $[x, I] = 0$,*