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INVARIANT SUBSPACES AND OPERATORS OF CLASS (S)

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Let $\mathscr{H}$ be an infinite dimensional separable complex Hilbert space, and let $\mathscr{L}(\mathscr{H})$ denote the algebra of all (bounded linear) operators on $\mathscr{H}$. This paper is concerned with a specific class of two-by-two operator matrices acting in the usual fashion on $\mathscr{H} \oplus \mathscr{H}$. An operator in $\mathscr{L}(\mathscr{H} \oplus \mathscr{H})$ will be said to be of class (S) if it can be represented as a two by two operator matrix of the form

$$\begin{bmatrix} A & V \\ -V^* & 0 \end{bmatrix}$$

where V is a unilateral shift of infinite multiplicity on $\mathscr{H}$ and A is an arbitrary operator in $\mathscr{L}(\mathscr{H})$.

In the present paper it is shown that the study of the operators of class (S) arises naturally in connection with the invariant subspace problem. In particular, the question of whether an operator of class (S) has a nontrivial invariant subspace is raised, and some significant results are obtained toward the solution of this problem.

Following [4] we shall denote by (F) the set of all operators which cannot be written in the form $\lambda + K$, where λ belongs to the complex field C, and K is in the ideal $\mathscr{K}$ of all compact operators. Brown and Pearcy in [4], Theorem 2 found, up to similarity, a standard form for operators in (F). As a consequence of that theorem they showed ([4], Corollary 3.4) that every operator $T \in (F)$ is similar to an operator matrix of the form

$$(*) \qquad \begin{bmatrix} R & W \\ S & 0 \end{bmatrix}$$

acting on $\mathscr{H} \oplus \mathscr{H}$, where W is an isometry of infinite deficiency (i.e. null W^* is infinite dimensional).

Our first objective in this paper is to obtain a simplification in the representing matrix $(*)$ (up to similarity) of an operator in (F). In this fashion, we prove (Theorem 1) that every operator $T \in (F)$ is similar to an operator matrix of the form

$$\begin{bmatrix} A & V \\ B & 0 \end{bmatrix},$$

where $A, B \in \mathscr{L}(\mathscr{H})$ and V is a unilateral shift of infinite multiplicity ([8]).