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# FIXED POINT THEOREMS FOR POINT-TO-SET MAPPINGS AND THE SET OF FIXED POINTS

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**Let $X$ be a Banach space and $K$ be a nonempty convex weakly compact subset of $X$. Belluce and Kirk proved that (1) If $f\colon K \to K$ is continuous, $\inf_{x \in K} \|x - f(x)\| = 0$ and $I - f$ is a convex mapping, then $f$ has a fixed point in $K$. (2) If $f\colon K \to K$ is nonexpansive and $I - f$ is a convex mapping on $K$, then $f$ has a fixed point in $K$. In this paper the concept of convex mapping has been extended to point-to-set mappings. Theorems 1 and 2 in § 2 extend the above fixed point theorems by Belluce and Kirk.**

**Let $W$ stand for the set of fixed points of $f\colon K \to cc(K)$. The set $W$ is called a singleton in a generalized sense if there is $x_0 \in W$ such that $W \subset f(x_0)$. In § 3 two examples are given to show that $W$ is not necessarily a singleton in a generalized sense if $f$ is strictly nonexpansive or if $I - f$ is convex. But one can be sure that $W$ is a convex set if $I - f$ is a convex or a semiconvex mapping.**

## 1. Preliminaries.

NOTATIONS AND DEFINITIONS. Let $X$ be a topological space, define

1. $2^X$ = the family of all nonempty closed subsets of $X$.
2. $b(X) = \{A \in 2^X;\ A \text{ is bounded}\}$, where $X$ is a metric space.
3. $k(X) = \{A \in 2^X;\ A \text{ is convex}\}$, where $X$ is a linear topological space.
4. $cpt(X) = \{A \in 2^X;\ A \text{ is compact}\}$.
5. $cc(X) = k(X) \cap cpt(X)$, where $X$ is a linear topological space.

In the remainder of this section we assume $X$ to be a metric space with metric $d$, unless otherwise stated.

6. Let $x \in X$ and $r > 0$, define $S(x, r) = \{y \in X;\ d(y, x) < r\}$.
7. For $x \in X$, $A \in 2^X$, define $d(x, A) = \inf\{d(x, y);\ y \in A\}$.
8. Given $A \in 2^X$ and $r > 0$, define $V_r(A) = \{x \in X;\ d(x, A) < r\}$.

LEMMA 1. *Let $x, y \in X$ and let $A$ be a nonempty subset of $X$. Then $d(x, A) \leqq d(x, y) + d(y, A)$.*

This is a simple consequence of the triangle inequality.

DEFINITION 1. Let $X$ be a topological space. A mapping