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ON NILPOTENCY AND RESIDUAL FINITENESS IN SEMIGROUPS

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It is proved that the class $\mathscr{C}$ of regular nilpotent semigroups coincides with the class of semilattices of nilpotent groups. Consequently, finitely generated semigroups in the class $\mathscr{C}$ are residually finite. The same results are true for semisimple 2-nilpotent semigroups.

1. Introduction. For semigroups defined in terms of generators and relations, the word problem is known to be recursively unsolvable in general (Post, [15]) but finitely presented semigroups which are residually finite do have a solvable word problem (McKinsey [13], T. Evans [3]). Although considerable work has been done to find large classes of residually finite groups (see e.g. the expository paper of W. Magnus [9]) only a few papers deal with residual finiteness in semigroups. Among the known results and apart from the solvability of the word problem, let us mention that any finitely generated residually finite semigroup is hopfian [4] and has a residually finite semigroup of endomorphisms [5]. Concerning classes of residually finite semigroups, one of the most significant results is due to A. I. Malcev who proved that finitely generated abelian semigroups are residually finite [12] (see also [1]). In trying to extend Malcev's result, one might recall an early result in group theory: Polycyclic, and in particular finitely generated nilpotent groups are residually finite (Hirsch [6]). A. I. Malcev [11], B. H. Neumann and Tekla Taylor [14] have shown that nilpotency of class c could be defined in group theory by the use of a law L_c not involving inverses. We shall recall the definition of L_c in the next section and adopt it as a definition of nilpotent semigroups. We then ask the following question:

Are finitely generated nilpotent semigroups residually finite? We show, (Corollary 3.1), that the answer is yes for finitely generated nilpotent regular (in the Von Neumann's sense) semigroups. Attempts to remove the regularity restriction in particular cases, (see Corollary 4.2) and examples, (see 4.5) lead us to consider that a positive answer to the question is not unreasonable. I am indebted to R. P. Hunter for drawing my attention to this problem.

2. Nilpotent semigroups. As in [14], we define the variety of nilpotent semigroups of class c inductively as follows: Let $q_1, q_2, \cdots, q_c$ be words in the variables $x, y, z_1, z_2, \cdots$, such that $q_1(x, y) = xy$ and