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NORMPRESERVING EXTENSIONS IN SUBSPACES OF $C(X)$

EGGERT BRIEM AND MURALI RAO

If B is a subspace of $C(X)$ and F is a closed subset of X, this note gives sufficient conditions in order that every function in the restriction subspace $B|_F$ has an extension in B with no increase in norm.

Introduction. Let X be a compact Hausdorff space, $C(X)$ the Banach algebra of all continuous complex-valued functions on X and let B be a closed linear subspace of $C(X)$ separating the points of X and containing the constants. A closed subset F of X is said to have the normpreserving extension property w.r.t. B if any function b_0 in the restriction subspace $B|_F$ has an extension $b \in B$ (i.e. $b|_F = b_0$) such that $||b|| = ||b_0||_F(||\cdot||$(resp. $||\cdot||_F$) denotes the supremum norm on X (resp. F)). The main result is the following:

Let F be a closed subset of X and suppose there is a map T (not necessarily linear) from $M(X)$ into $M(X)$ satisfying the following conditions

(i) $m - Tm \in B^\perp$ for all $m \in M(X)$

(ii) $T\lambda$ is a probability measure when λ is

(iii) If $s_i \in C$ and $m_i \in M(X)$ $i = 1, \cdots, n$ and $\sum_{i=1}^n s_i m_i \in k(F)^\perp$ then $\sum_{i=1}^n s_i(Tm_i)|_{X\setminus F} \in B^\perp$.

Then F has the normpreserving extension property.

$M(X)$ denotes the set of regular Borel measures on X, and if A is a subset of B then $A^\perp$ is the set of those measures in $M(X)$ which annihilate A. $k(F)$ consists of those functions in B which are identically 0 on F. Also if G is a Borel subset of X and $m \in M(X)$ then $m|_G$ is the measure $\chi_G m$ where χ_G is the characteristic function for G.

Two conditions, either of which is known to imply that a closed subset F of X has the normpreserving extension property are the following:

Condition 1. *For all $\sigma \in B^\perp$, $\sigma|_F \in B^\perp$.*

Condition 2. *F is a compact subset of the Choquet boundary Σ_B for B and for all $\sigma \in M(\Sigma_B) \cap B^\perp$, $\sigma|_F \in B^\perp$.*

($M(\Sigma_B)$ denotes the set of those $\sigma \in M(X)$ for which the total variation $|\sigma|$ is maximal in Choquet's ordering for positive measures (see [1]