## ON GENERALIZATIONS OF SYLOW TOWER GROUPS

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In this paper two different generalizations of Sylow tower groups are studied. In Chapter I the notion of a k-tower group is introduced and a bound on the nilpotence length (Fitting height) of an arbitrary finite solvable group is found. In the same chapter a different proof to a theorem of Baer is given; and the list of all minimal-not-Sylow tower groups is obtained.

Further results are obtained on a different generalization of Sylow tower groups, called Generalized Sylow Tower Groups (GSTG) by J. Derr. It is shown that the class of all GSTG's of a fixed complexion form a saturated formation, and a structure theorem for all such groups is given.

## NOTATIONS

The following notations will be used throughout this paper:

| $N \triangleleft G$  | N is a normal subgroup of $G$                |
|--|--|
| $N\operatorname{Char} G$                                   | N is a characteristic subgroup of $G$        |
| $N \cdot \triangleleft G$                                  | N is a minimal normal subgroup of $G$        |
| M < G  | M is a proper subgroup of $G$                |
| $\mathit{M} < \cdot \mathit{G}$                            | M is a maximal subgroup of $G$               |
| $Z\!(G)$   | the center of $G$                            |
| $ G _p$  | p-part of the order of $G$ , $p$ a prime     |
| $\pi(G)$   | set of all prime divisors of $ G $           |
| $\phi(G)$  | the Frattini subgroup of $G$ = the intersec- |
|  | tion of all maximal subgroups of $G$         |
| [H]K   | semi-direct product of $H$ by $K$            |
| F(G)   | the Fitting subgroup of $G$ = the maximal    |
|  | normal nilpotent subgroup of $G$             |
| $\mathit{C}(H) = \mathit{C}_{\mathit{G}}(H)$               | the centralizer of $H$ in $G$                |
| $N(H) = N_{\scriptscriptstyle G}(H)$                       | the normalizer of $H$ in $G$                 |
| $P \in \operatorname{Syl}_p(G)$                            | P is a Sylow $p$ -subgroup of $G$            |
| $P$ is a $S_p$ -subgroup of $G$                            | $P \in \operatorname{Syl}_p(G)$              |
| $\mathrm{Core}(H)=\mathrm{Core}_{\scriptscriptstyle G}(H)$ | the largest normal subgroup of $G$ contained |
|  | in $H = \bigcap_{g \in G} H^g$               |
| l(G)   | the nilpotence length (Fitting height) of G  |
| $l_p(G)$   | p-length of $G$                              |
| d(G)   | minimal number of generators of $G$          |
| c(P)   | nilpotence class of the $p$ -group $P$       |
| $p^*$  | some nonnegative power of prime p            |
| $O_p(G)$   | largest normal $p$ -subgroup of $G$          |