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FIXED POINT SETS

L. E. WARD, JR.

The principal results are the following. If M is a metric space homeomorphic to a subset of a real linear space which is star-shaped with respect to an element p, or if M is homeomorphic to an arcwise connected subspace of a dendroid which is smooth at a point p, then each closed subset of M which contains p is the fixed point set of a continuous mapping of M. If M is a continuum having Property W (this is a class of Peano continua containing the local dendrites and the continua containing no continuum of condensation) then each nonempty closed subset of M is a fixed point set. It is shown that a subset K of a dendrite is the fixed point set of a continuous surjection if and only if the complement of K is not homeomorphic to $[0, \infty)$.

1. Introduction. This paper is concerned with the following problem. *If M is a topological space and if F is a subset of M, when does there exist a continuous mapping $f: M \to M$ such that F is precisely the set of fixed points of f?* If M has the fixed point property then it is obvious that F must be nonempty, and if M is a Hausdorff space then F must be a closed set. Moreover, M and each singleton subset of M are the fixed point sets of the identity and constant mappings, respectively.

Let us call the subset F of M a *fixed point set of M* if there exists a continuous self-mapping of M whose set of fixed points is exactly F. The space M has the *complete invariance property* if each of its nonempty closed subsets is a fixed point set. Relatively little information concerning the fixed point sets of a space has appeared in the literature. H. Robbins [7] has shown by a very simple argument that an n-cell has the complete invariance property, and Helga Schirmer [9] proved that a dendrite has the complete invariance property. In what follows we are able to widen somewhat the class of spaces which have this property and we also determine all of the subsets of a dendrite which are the fixed point sets of a continuous surjection. However, the following intriguing question remains open. *Does every Peano continuum have the complete invariance property?*

2. Fixed point sets of dendrites and dendroids. The arguments employed by Robbins and Schirmer made implicit use of the potent contractibility properties of n-cells and dendrites. Our first result isolates the precise properties they exploited.