

CENTRALIZERS OF TWISTED GROUP ALGEBRAS

ROBERT C. BUSBY

Centralizers (left, right, and double) of rings and algebras have received much attention recently, and seem likely to become an important topic in ring theory. They have proved quite useful in Banach algebra theory, and a good deal of work has been done on the computation of centralizers for various Banach algebras. In this paper we compute the left centralizers of a twisted group algebra, a generalization of the group algebra of locally compact group, which includes as special cases the covariance algebras of quantum field theory, and the group algebras of separable group extensions (explicitly given in terms of the subgroup algebra and quotient group). We give a representation of the algebra of left centralizers of a "locally continuous" twisted group algebra as an algebra of vector-valued measures with "twisted convolution". This result gives more than explicit computation of centralizers. The form of the result enables us to investigate isometric isomorphisms between twisted group algebras along lines previously pursued for ordinary group algebras. In some cases we get a complete description of possible isomorphism classes in terms of orbits in a cohomology set.

Double Centralizers were first introduced by G. Hochschild in his cohomology studies [15], and later, independently, by B. E. Johnson in [16] (see also [17]), who was largely interested in applications to analysis. Work on the ideal theory of algebras and Banach algebras by Dauns and Hofmann ([6], [7]), and on extensions of C^* -algebras [1] are examples of such applications. Left centralizers are used in the papers of Wendel on norm decreasing isomorphisms of group algebras ([21], [22]). The latter results are especially important and suggestive for us. Wendel showed in [21] Th. 2 that the left centralizers of a group algebra form an algebra isometrically isomorphic with the algebra $M(G)$ of complex valued, bounded, regular Borel measures on G . This suggested the result given above, namely that centralizers of a twisted group algebra should be vector-valued measures. We also show, in analogy to a result of Wendel, that the isometric left centralizers correspond to certain measures with a one point support. Moreover, several of our arguments take those of Wendel as a starting point. The description of isometric isomorphisms of group algebras given in [21] is the forerunner of our results on classifying isometric isomorphisms of twisted group algebras, and, as a matter of fact, is a corollary of them. The form of our results on