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PSEUDO-COMPLETENESS AND THE PRODUCT OF BAIRE SPACES

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The class of pseudo-complete spaces defined by Oxtoby is one of the largest known classes $\mathscr{C}$ with the property that any member of $\mathscr{C}$ is a Baire space and $\mathscr{C}$ is closed under arbitrary products. Furthermore, all of the classical examples of Baire spaces belong to $\mathscr{C}$. In this paper it is proved that if $X \in \mathscr{C}$ and if Y is any (quasi-regular) Baire space, then $X \times Y$ is a Baire space. The proof is based on the notion of A-embedding which makes it possible to recognize whether a dense subspace of a Baire space is a Baire space in its relative topology. Finally, examples are presented which relate pseudo-completeness to several other types of completeness.

1. **Introduction.** A space X is a *Baire space* if every nonempty open subset is of second category [2] or, equivalently, if the intersection of countably many dense open subsets of X is dense in X. Locally compact Hausdorff spaces and completely metrizable spaces are the classical examples of Baire spaces.

In [10] Oxtoby introduced the notion of a pseudo-complete space (see §2 for precise definitions). Pseudo-complete spaces are Baire spaces and the classical examples of Baire spaces are pseudo-complete. Also, Čech-complete spaces (i.e., G_δ-subsets of compact Hausdorff spaces [3]) as well as subcompact spaces [6] belong to the class of pseudo-complete spaces.

Pseudo-completeness has nice invariance properties. In particular, the topological product of any family of pseudo-complete spaces is pseudo-complete. Thus such a product is a Baire space.

In dealing with pseudo-completeness, assumptions about the usual separation axioms are irrelevant. However, it is often convenient to consider spaces which are *quasi-regular*, i.e., every nonempty open set contains the closure of some nonempty open set (cf. [10]).

Oxtoby [10] has also given an example of a completely regular Baire space whose square is not a Baire space, thus showing that a product theorem for Baire spaces cannot be obtained without some additional condition on (at least one of) the factors.

The main result of our paper is that the product of a quasi-regular Baire space and a pseudo-complete space is a Baire space. The techniques employed here, especially those in §4, are quite different from the usual category type techniques.

This paper is organized as follows. In §2 we discuss some new