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SLENDER RINGS AND MODULES

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An abelian group B is called slender if every homomorphism from $\prod_1^\infty Z$ into B is continuous for the discrete topology on B and the topology that $\prod_1^\infty Z$ has as a cartesian product if Z (the additive group of integers) is considered discrete. It can be seen that if B is slender, then every homomorphism from $\prod_1^\infty A_i$ into B is continuous for the same topologies, where A_i are arbitrary abelian groups. Slender groups were completely characterized by Nunke, and it follows from his characterization that all countable reduced torsion free groups are slender. Nunke's results depend on the fact that Z is slender (proved by Specker) and the use of some homological machinery. In this paper, slender modules are studied over an arbitrary ring. A ring R will be called slender if R is a slender R-module. In Theorem 1, a generalization of the Baire Category Theorem is used to show that a countable module over an arbitrary ring is slender if $\bigcap \mathfrak{m}B = 0$, where $\mathfrak{m}$ ranges over the set of ideals in R which are not zero divisors for B. It follows that countable torsion free reduced modules over any (countable) integral domain are slender. In Theorem 2, it is shown that a commutative ring R is slender if there exists an infinite set of maximal ideals with the property that the intersection of any infinite subset is 0.

In § 2, reflexive modules over a slender ring are studied, and it is shown that a projective module is reflexive if it is generated by a set with cardinality smaller than the first measurable cardinal. In § 3, Nunke's characterization is extended to modules over a slender Dedekind domain having only countably many ideals. The general approach in § 3 follows Nunke's, but all homological machinery is avoided by the use of topological techniques.

R will always be a ring with identity. The word ideal will mean a two sided ideal, the word module will mean a unitary left module.

If B is an R-module and $\mathfrak{m}$ an ideal in R, we say that $\mathfrak{m}$ is a zero divisor for B if $\mathfrak{m}x = 0$ for some nonzero $x \in B$. We say that B is torsion free if no nonzero ideal is a zero divisor for B. If $\mathscr{M}$ is a family of ideals in R, then by the $\mathscr{M}$-adic topology on B we mean the linear topology induced by the family of submodules $\mathfrak{m}B$ where $\mathfrak{m}$ is a finite intersection of ideals in $\mathscr{M}$. The word complete will only be used in reference to Hausdorff topologies.