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ON RELATIONS BETWEEN NÖRLUND AND RIESZ MEANS

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Several results on relations between (absolute) Nörlund summability and (absolute) Riesz summability are known. Among them, Dikshit gives sufficient conditions for $|\bar{N}, q_n| \subseteqq |N, p_n|$ when the sequence $\{p_n\}$ is nonincreasing. The purpose of this paper is to give sufficient conditions for $|N, p_n| \subseteqq |\bar{N}, q_n|$ or $|\bar{N}, q_n| \subseteqq |N, p_n|$ when $\{p_n\}$ is monotone. The results obtained here are also absolute summability analogues of Ishiguro's theorems and Kuttner and Rhoades' theorems which state the inclusion relations between (N, p_n) and $(\bar{N}, p_n)$ summability.

1. Let $\{p_n\}$ be a sequence such that $p_n > 0$, $P_n = \sum_{k=0}^{n} p_k \neq 0$. A series $\sum_{n=0}^{\infty} a_n$ with its partial sum s_n is said to be summable (N, p_n) to sum s, if $t_n = \sum_{k=0}^{n} p_{n-k}s_k/P_n \to s$ as $n \to \infty$, and summable $(\bar{N}, p_n)$ to sum s, if $u_n = \sum_{k=0}^{n} p_k s_k/P_n \to s$ as $n \to \infty$. It is said to be absolutely summable (N, p_n), or summable $|N, p_n|$, if $\Sigma|t_n - t_{n+1}| < \infty$, and absolutely summable $(\bar{N}, p_n)$, or summable $|\bar{N}, p_n|$, if $\Sigma|u_n - u_{n+1}| < \infty$. Given two summability methods A and B, we write $(A) \subseteqq (B)$ if each series summable A is summable B. Throughout this paper, we write for a sequence $\{b_n\}$

$$b_{-n} = 0 (n \geqq 1),\ \Delta b_n = b_n - b_{n+1}\ ,$$

and for a double sequence $\{c_{mn}\}$

$$\Delta_n(c_{mn}) = c_{mn} - c_{m,n+1}\ ,$$

and K denotes an absolute constant, not necessarily the same at each occurrence.

On inclusion relations between two summability, the following results are known.

THEOREM A. [1] *If the sequence $\{p_n\}$ is nonincreasing, $Q_n \to +\infty$ and $Q_n/q_{n+1} = O(P_{n+1})$, where $q_n > 0$ and $Q_n = \sum_{k=0}^{n} q_k \neq 0$, then $|\bar{N}, q_n| \subseteqq |N, p_n|$.*

THEOREM B. [2] *If $\{p_n\}$ is the nondecreasing sequence such that $P_n \to +\infty$ and $p_n = o(P_n)$, then $(\bar{N}, p_n) \subseteqq (N, p_n)$.*

THEOREM C. [3] *If $\{p_n\}$ is the nonincreasing sequence such that $P_n \to +\infty$, then $(N, p_n) \subseteqq (\bar{N}, p_n)$.*