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THE ALGEBRA OF BOUNDED CONTINUOUS FUNCTIONS INTO A NONARCHIMEDEAN FIELD

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Let S be a topological space, F a complete nonarchimedean rank 1 valued field, and $C^*(S, F)$ the Banach algebra of bounded, continuous, F-valued functions on S. Various topological conditions on S and/or F are shown to be equivalent, respectively, to each of the following: every maximal ideal of $C^*(S, F)$ is fixed; the only quotient field of $C^*(S, F)$ is F itself; every homomorphism of $C^*(S, F)$ into F is an evaluation at a point of S; the Stone-Weierstrass theorem holds for $C^*(S, F)$. It is also shown that a certain topological space derived from S may be embedded in the space of maximal ideals of $C^*(S, F)$ with Gelfand topology, or in the space of homomorphisms of $C^*(S, F)$ into F.

0. Introduction. Throughout this paper, $C^*(S, F)$ denotes the Banach algebra of bounded, continuous functions on a topological space S into a complete nonarchimedean rank 1 valued field F. We introduce several stronger-than-usual topological separation properties, such as ultrahausdorff, ultraregular, and ultranormal; and several weaker-than-usual compactness properties, such as mildly compact, mildly countably compact, and mildly Lindelof. We then show that several key implications involving $C^*(S, F)$ become equivalences when the new topological properties replace their conventional counterparts.

In §1, we define and discuss these new topological properties, and relate them to the cofilters ("ouf-filtres") of van der Put [13]. In §2, we obtain a result on the metric structure of non-locally compact nonarchimedean Banach spaces.

In §3, we show that all maximal ideals of $C^*(S, F)$ are fixed if and only if S is mildly compact (Theorem 15); and that F is the only quotient field of $C^*(S, F)$ if and only if F is locally compact or S is mildly countably compact (Theorem 19). Using the result of §2, we also give necessary and/or sufficient conditions for the only homomorphisms of $C^*(S, F)$ into F to be evaluations at points of S (Theorems 20 and 21). We also show that the set of quasicomponents of S, appropriately topologized, is homeomorphic to the space of fixed maximal ideals of $C^*(S, F)$, with either of the Gelfand topologies defined by Shilkret [14] (Theorems 10 and 12).

In §4, we extend results of Kaplansky [7] and Chernoff, Rasala, and Waterhouse [3]: we introduce two versions of the Stone-Weierstrass property, and show that the stronger version in $C^*(S, F)$ is equivalent to mild compactness of S, and the weaker version is sufficient for mild