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HOMEOMORPHISMS OF MANIFOLDS WITH ZERO-DIMENSIONAL SETS OF NONWANDERING POINTS

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Let h be a self-homeomorphism of a compact n-dimensional manifold M, which is not homeomorphic to an odd dimensional sphere, such that the set N of irregular points of h is closed in M and the set of nonwandering points of h is zero-dimensional. One of the main results of this paper is that N is either connected or consists of the two fixed points of h. In the latter case, N is homeomorphic to the n-sphere. In the former case when $n = 2$, it is shown that each component of $M - N$ is an open 2-cell. If M is open and N is compact, then it is shown that M is homeomorphic to Euclidean n-space and N consists of a single fixed point of h.

Let h be a self-homeomorphism of a metric space X with metric d. h is *regular* at $x \in X$ if, for every $\varepsilon > 0$, there exists $\delta > 0$ such that whenever $y \in X$ and $d(x, y) < \delta$, then $d(h^n(x), h^n(y)) < \varepsilon$ for all integers n. h is *irregular* at $x \in X$ if h is not regular at x. Let $E(X, h)$ and $N(X, h)$ denote the set of regular and irregular points, respectively of X. This definition was originally given by Kerékjártó [10]. If X is locally compact and connected, it was shown by Homma and Kinoshita [7] that if $N(X, h)$ is a finite set which does not separate X, then $N(X, h)$ consists of the fixed points of h and contains at most two points. Kaul [9] obtained a similar result with the assumption that X be locally connected and $N(X, h)$ be compact zero-dimensional and not separating X. In [12], [14] Lam obtained analogous results when $N(X, h)$ need not be zero-dimensional and $E(X, h)$ has a property called indivisibility. The property of indivisibility is generally weaker than the property that $E(X, h)$ be connected. As a result of [12], Kaul's theorem can be shown to be valid without assuming that X be locally connected (cf. Theorem 2.11 in the following); hence, Kaul's theorem is a generalization of the theorem of Homma and Kinoshita. If X is an open connected manifold, $N(X, h)$ is a compact set not separating X and if h is positively regular on all of X, then it was shown by Duvall and Husch [4] that $N(X, h)$ is a strong deformation retract of X and, hence, must be connected.

A point $x \in X$ is *nonwandering* or *regionally recurrent* under h, if for every neighborhood U of x there exists a subsequence $\{m_i\}_{i=1}^{\infty}$ of the positive integers such that $h^{m_i}(U) \cap U \neq \varnothing$; let $R(X, h)$ be the set of all nonwandering points of h in X. One of the steps in Homma