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THE NUMBER OF MULTINOMIAL COEFFICIENTS DIVISIBLE BY A FIXED POWER OF A PRIME

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In this paper some results of L. Carlitz and the writer concerning the number of binomial coefficients divisible by p^j but not by p^{j+1} are generalized to multinomial coefficients. In particular $\theta_j(k; n)$ is defined to be the number of multinomial coefficients $n!/n_1!, \cdots, n_k!$ divisible by exactly p^j, and formulas are found for $\theta_j(k; n)$ for certain values of j and n. Also the generating function technique used by Carlitz for binomial coefficients is generalized to multinomial coefficients.

1. Introduction. Let p be a fixed prime and let n and j be nonnegative integers. L. Carlitz [2], [3] has defined $\theta_j(n)$ as the number of binomial coefficients

$$\binom{n}{r} \qquad (r = 0, 1, \cdots, n)$$

divisible by exactly p^j and he has found formulas for $\theta_j(n)$ for certain values of j and n. In particular, if we write

$$n = a_0 + a_1 p + \cdots + a_s p^s \qquad (0 \leqq a_i < p) \tag{1.1}$$

then

$$\theta_0(n) = (a_0 + 1)(a_1 + 1) \cdots (a_s + 1)$$

$$\theta_1(n) = \sum_{i=0}^{s-1} (a_0 + 1) \cdots (a_{i-1} + 1)(p - a_i - 1)a_{i+1}(a_{i+2} + 1) \cdots (a_s + 1) .$$

The writer [5], [6] has also considered the problem of evaluating $\theta_j(n)$.

The purpose of this paper is to consider the analogous problem for multinomial coefficients and to generalize some of the formulas developed by Carlitz and the writer. Thus we define $\theta_j(k; n)$ as the number of multinomial coefficients

$$(n_1, \cdots, n_k) = \frac{n!}{n_1! \cdots n_k!} (n_1 + \cdots + n_k = n)$$

divisible by exactly p^j. In this definition the order of the terms $n_1, \cdots, n_k$ is important. We are distinguishing, for example, between (1, 2, 3) and (2, 1, 3). Clearly $\theta_j(2; n) = \theta_j(n)$.

In this paper we find formulas for $\theta_0(k; n)$, $\theta_1(k; n)$, and $\theta_2(k; n)$. We also show how the generating function method used by Carlitz