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REAL ANALYTIC OPEN MAPS

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Let R and C be the real and complex fields, respectively, and for $\zeta \in C$ let $\mathscr{R}(\zeta)$ be the real part of ζ. If $f: M^{p+1} \to N^p$ is real analytic and open with $p \geqq 1$, then there is a closed subspace $X \subset M^{p+1}$ such that $\dim f(X) \leqq p-2$ and, for every $x \in M^{p+1} - X$, there is a natural number $d(x)$ with f at x locally topologically equivalent to the map

$$\phi_{d(x)}: C \times R^{p-1} \longrightarrow R \times R^{p-1}$$

defined by $\phi_{d(x)}(z, t_1, \cdots, t_{p-1}) = (\mathscr{R}(z^{d(x)}), t_1, \cdots, t_{p-1})$.

In [7] Nathan proved: If $f: M^2 \to N^1$ is real analytic and open, then for every $x \in M^2$ there is a natural number $d(x)$ with f at x locally topologically equivalent to the map $\phi_{d(x)}: C \to R$ defined by $\phi_{d(x)}(z) = \mathscr{R}(z^{d(x)})$. This is the case $p = 1$ of the above theorem, but our proof is not a generalization of his.

Examples (see (3.3)) show that "topologically equivalent" cannot be replaced by "analytically equivalent" or even "C^1 equivalent", f real analytic cannot be replaced by f C^∞ (but see (3.1)), an exceptional set X with $\dim f(X) \geqq p-2$ is needed, and $\dim X$ may be $p-1$.

CONVENTIONS 1.2. We must assume that the reader has [2] at hand, and we follow its conventions. In particular we need [2; (2.2), (2.4), (2.5), (2.6), (2.8), (2.9), (3.1), and (3.9)]. For $f: M^n \to N^p$, B_f is the set of x in M^n at which f is not locally topologically equivalent to the projection map $\rho: R^n \to R^p$. The symbol $\approx$ is read "is diffeomorphic to".

DEFINITIONS 1.3. C-analytic sets are defined in [2]. A C-analytic set is called *C-irreducible* [9, p. 155] if it is not the sum of two C-analytic subsets distinct from itself. Whitney and Bruhat [9, p. 155, Proposition 11] prove that any C-analytic set V is uniquely the (countable) locally finite union of C-irreducible C-analytic subsets V_m, no one of which contains another. The V_m are called the *C-irreducible components* of V. Conversely, any locally finite union of C-analytic sets is a C-analytic set [9, p. 154].

DEFINITIONS 1.4. Let V be a complex analytic set of dimension v. There is a complex analytic subset $S \subset V$ such that $\dim S < v$ and $V - S$ is a complex analytic v-manifold [8, p. 500]. (The points of